Asteroid Noise in the Motions of the Inner Planets

Preliminary Results

Marc A. Murison

Astronomical Applications Department
U.S. Naval Observatory

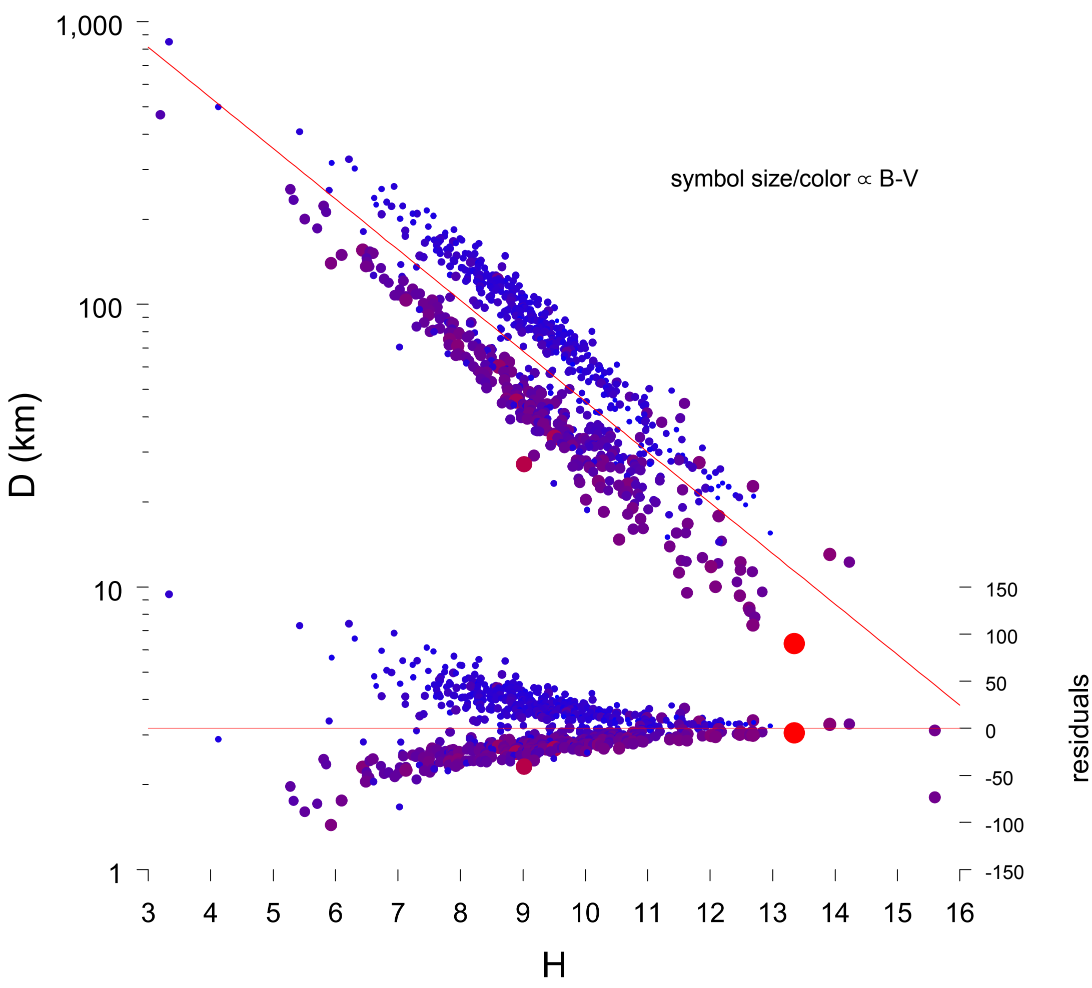
DDA Yosemite
April 10, 2000

Motivation

- Radar ranging from spacecraft has given us Earth-Mars distance measurements at the several-meter level
- Largest source of planetary ephemeris uncertainty: unmodeled perturbations from asteroids
- Previous studies (Mars)
  - Goal: to determine asteroid masses using perturbations of Mars
  - e.g.: Williams (1984): analytical study
  - e.g.: Standish & Hellings (1989): Viking lander data
- Flip the goal: what is the effect of asteroid perturbations on the planetary motions?
- A Different Perspective: view the asteroids as a noise source
- A complicated dynamical system
  - Direct perturbations
  - Indirect perturbations
  - 2nd order indirect perturbations...

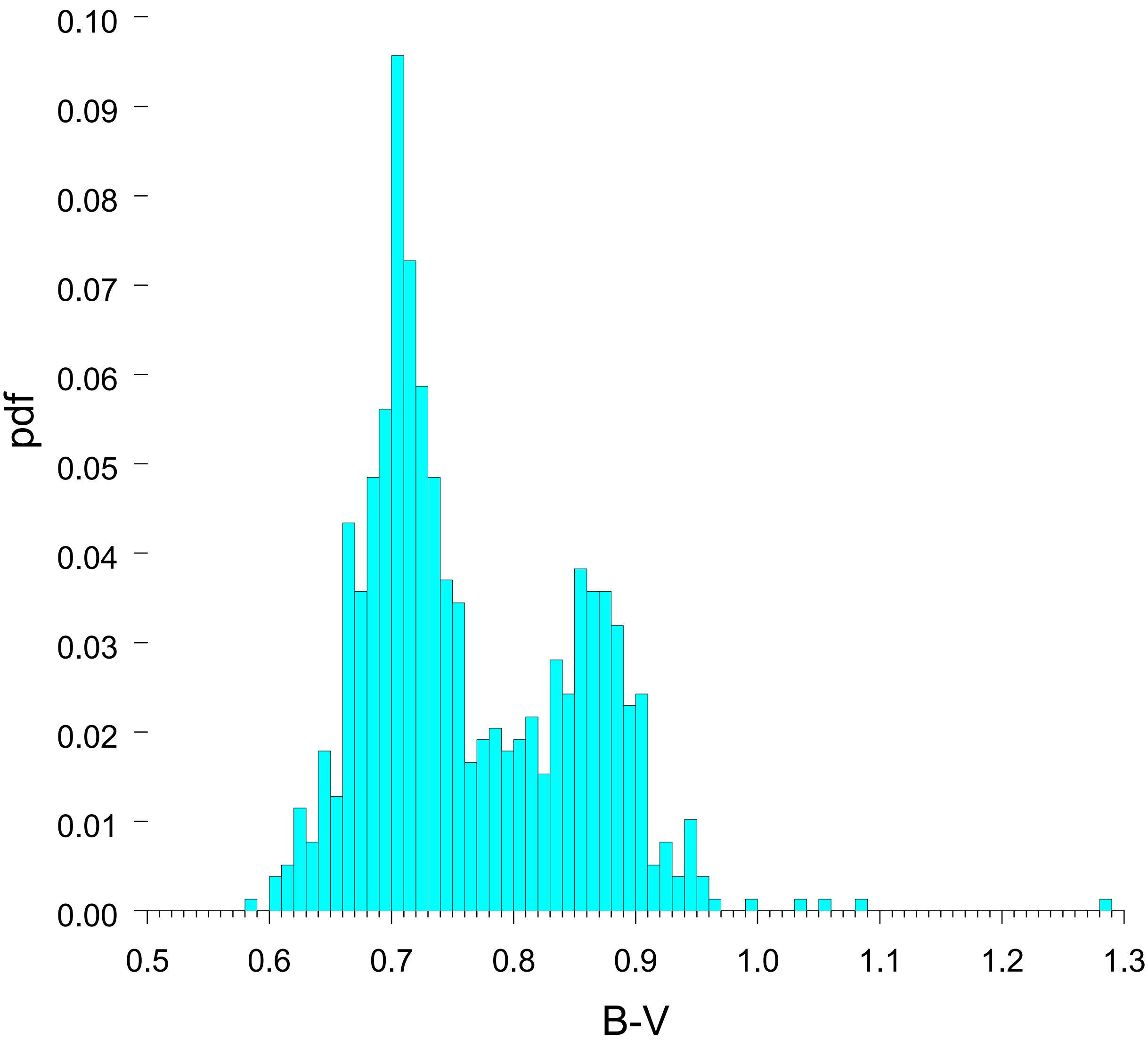
Asteroid Mass — An Interesting Subject

- Largest asteroid masses taken from recent data
  - Hilton (1999), and Hilton (private comm., see http://aa.usno.navy.mil/hilton/asteroid_masses.htm)
- Other masses estimated from fit to IRAS diameters
  - Bimodality of distribution ignored
  - Assume spherical bodies and ρ = 3.0 g/cm^3
  - On average, masses are likely overestimated



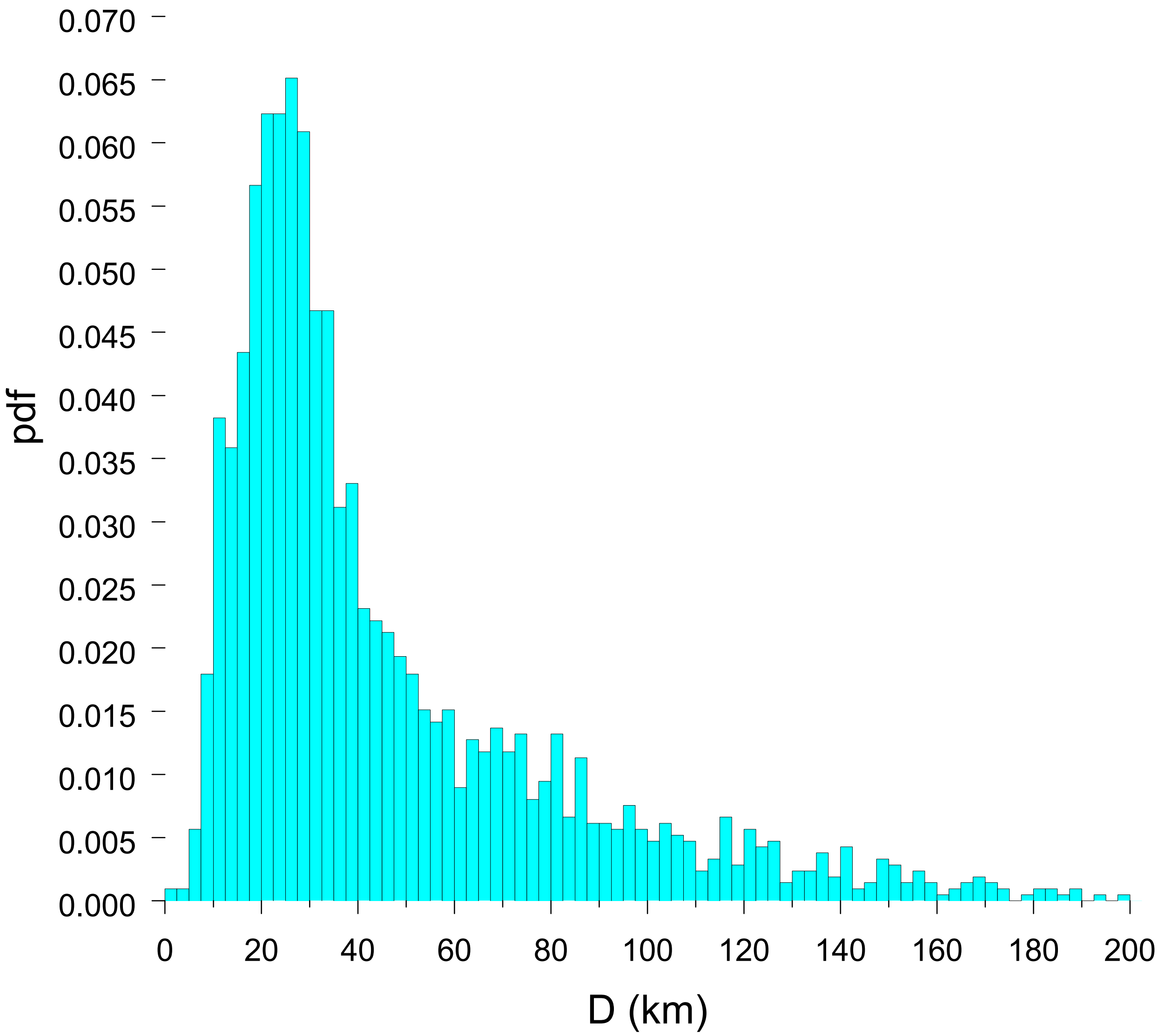
Asteroid Color Distribution

- Probability density function of 2,144 IRAS asteroids as a function of B-V
- Note: distribution is bimodal



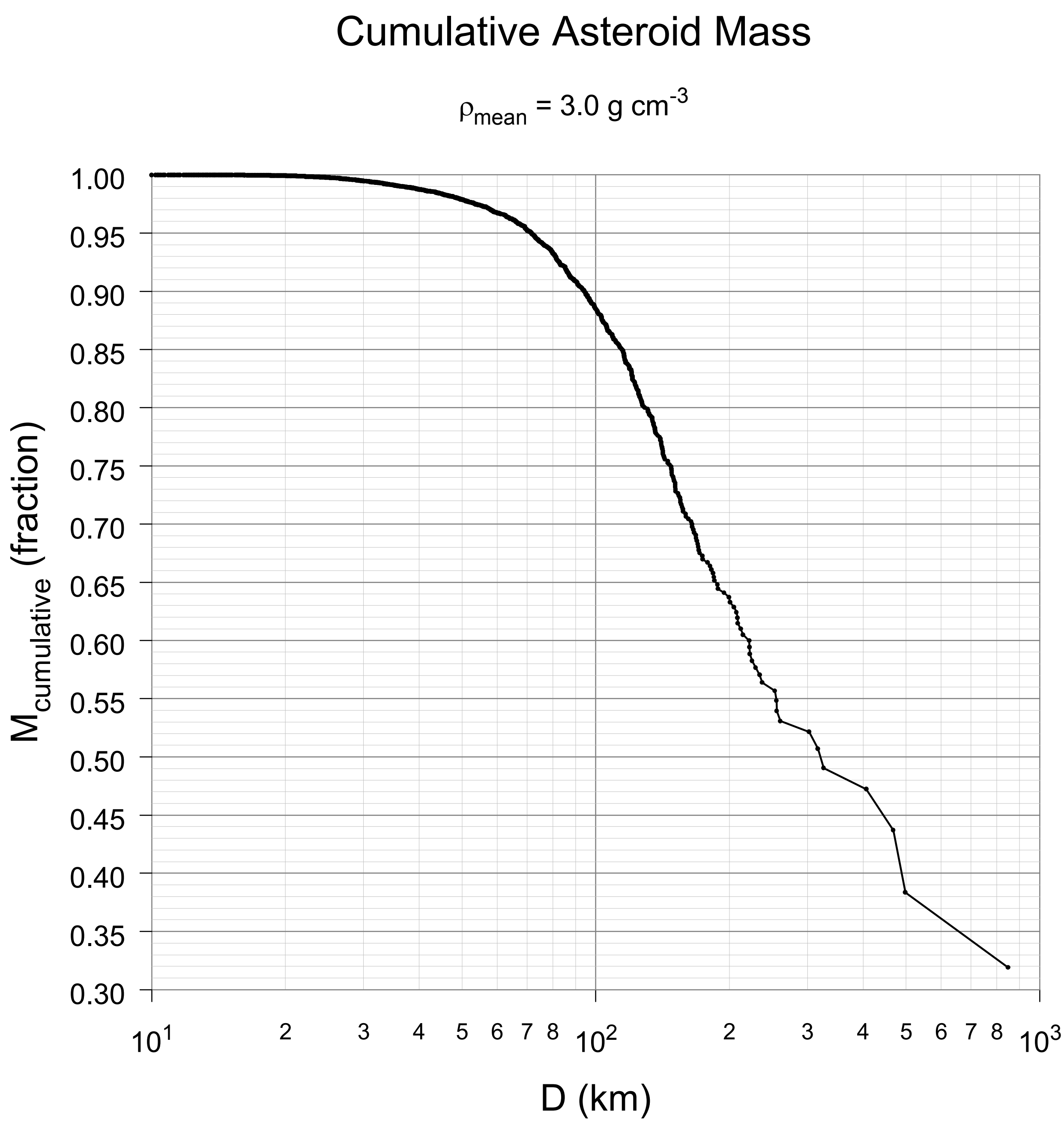
Asteroid Diameter Distribution

- Probability distribution function of 2,144 IRAS asteroids as a function of estimated diameter
  - Cutoff at low end is due to observational bias



Cumulative Asteroid Mass Distribution

- D > 85 km ~ 92% of total mass represented by asteroids having IRAS diameters (2,144)



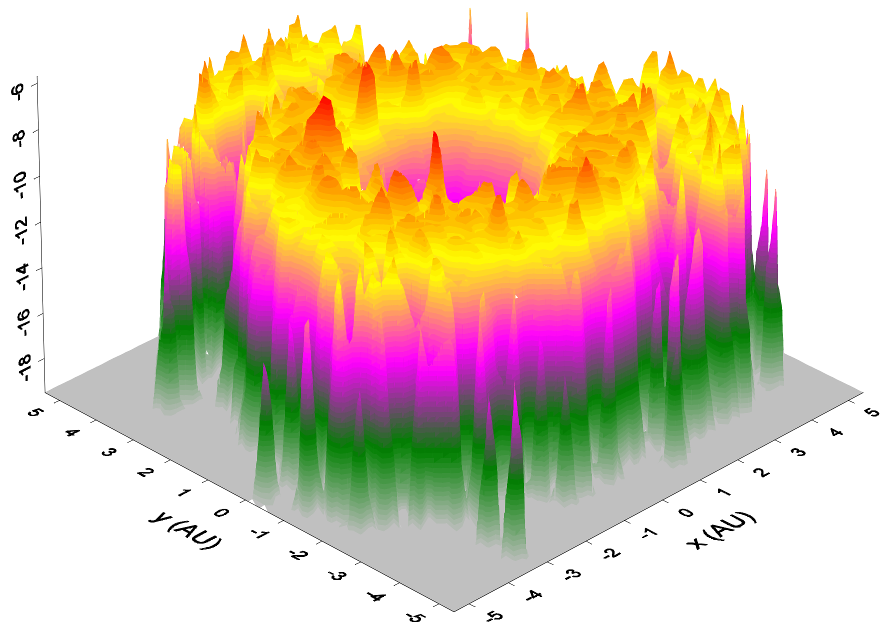
.

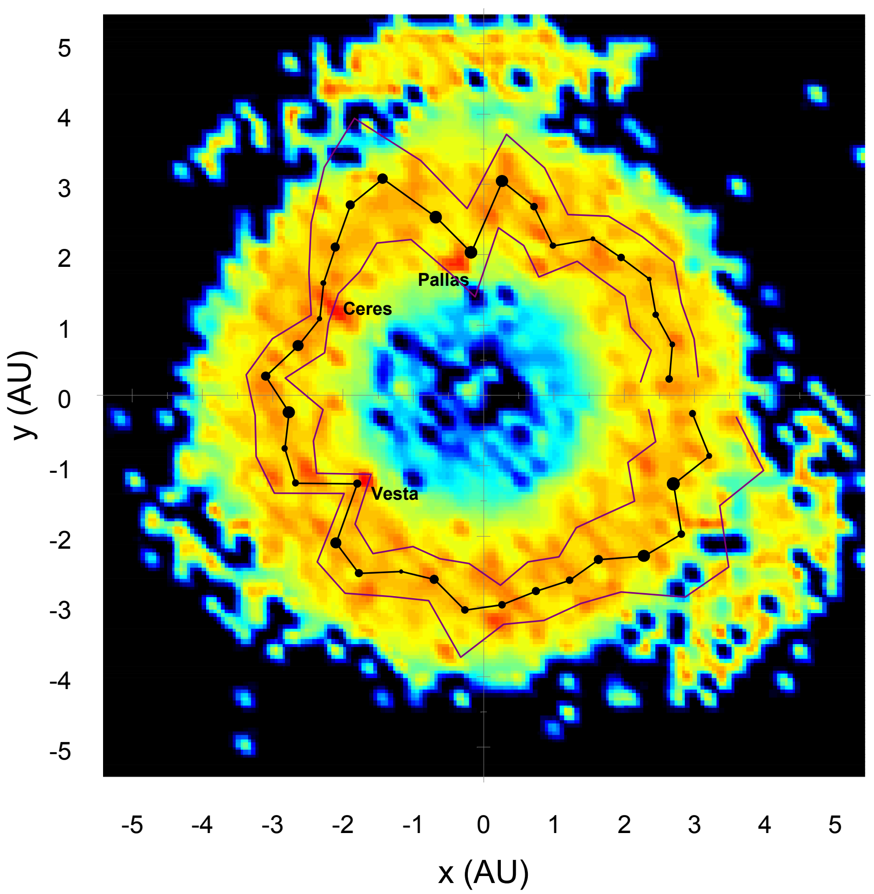
The 25 Largest Asteroids

number	name	D (km)	M (10^{-6} M_E)
1	Ceres	848.4	126.89
2	Pallas	498.1	25.68
4	Vesta	468.3	21.34
10	Hygiea	407.1	14.02
511	Davida	326.1	7.21
704	Interamnia	316.6	6.59
52	Europa	302.5	5.75
87	Sylvia	260.9	3.69
31	Euphrosyne	255.9	3.48
15	Eunomia	255.3	3.46
16	Psyche	253.2	3.37
65	Cybele	237.3	2.78
3	Juno	233.9	2.66
324	Bamberga	229.4	2.51
451	Patientia	225.0	2.37
107	Camilla	222.6	2.29
532	Herculina	222.2	2.28
48	Doris	221.8	2.27
45	Eugenia	214.6	2.05
29	Amphitrite	212.2	1.99
9	Metis	(211.1)	1.96
14	Irene	(209.4)	1.91
121	Hermione	209.0	1.90
423	Diotima	208.8	1.89
13	Egeria	207.6	1.86

Spatial Mass Distribution: All Asteroids

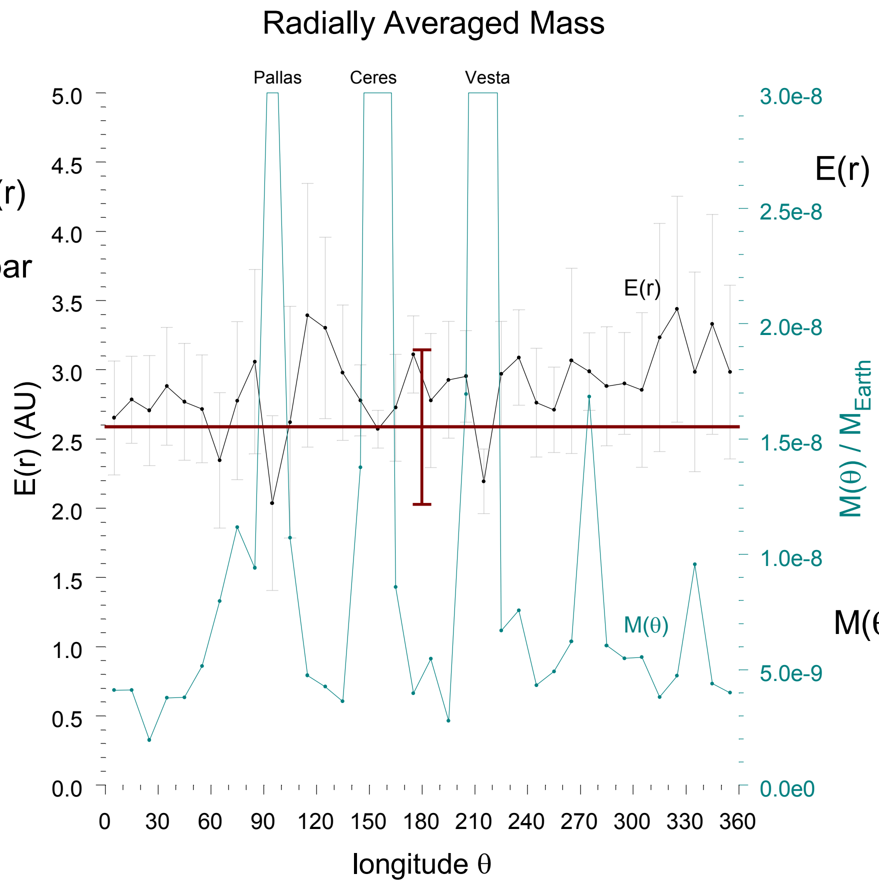
- Log of mass density (Earth masses per AU^2)





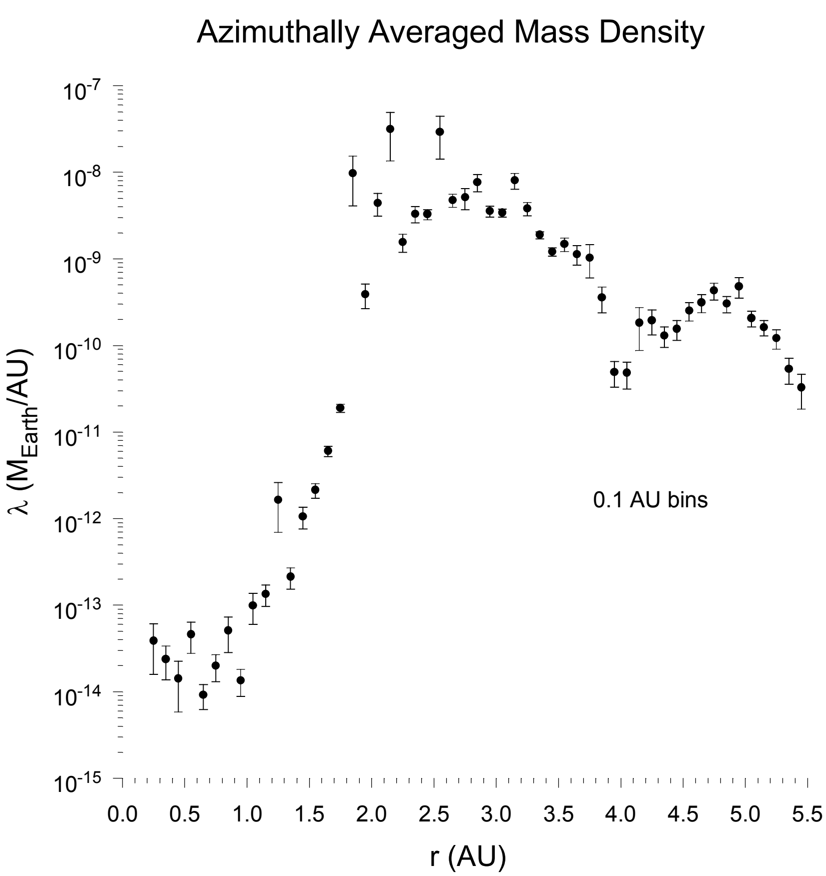
Spatial Mass Distribution: All Asteroids

Center line is E(r) averaged over azimuth (error bar is 1 σ)



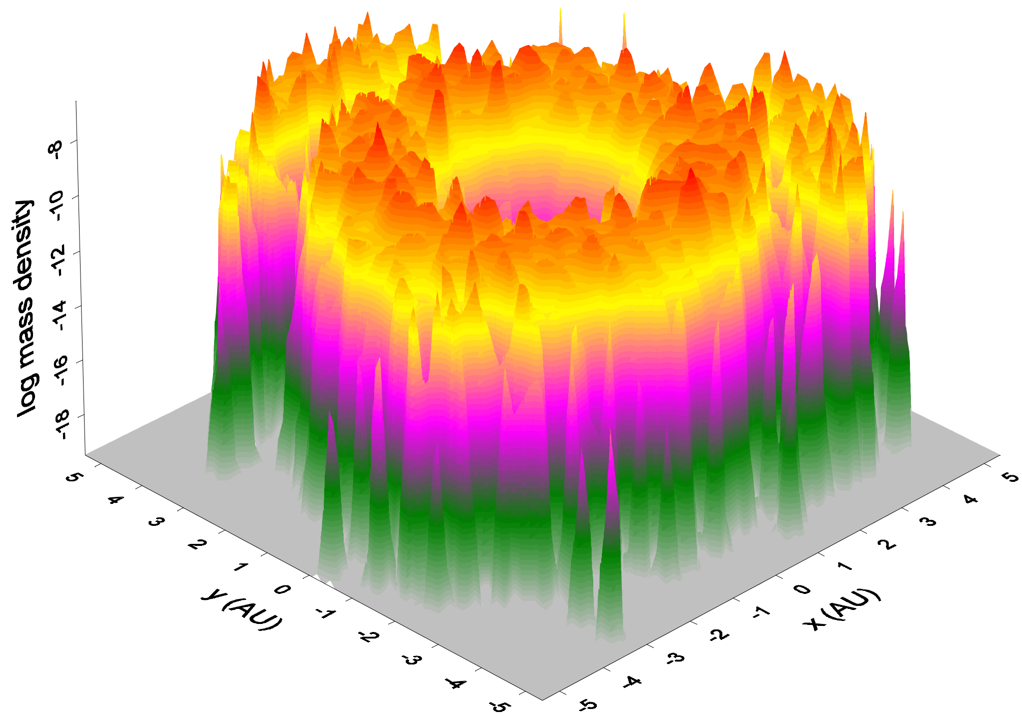
E(r) = radial mass density weighted expectation value (with 1-σ errors)

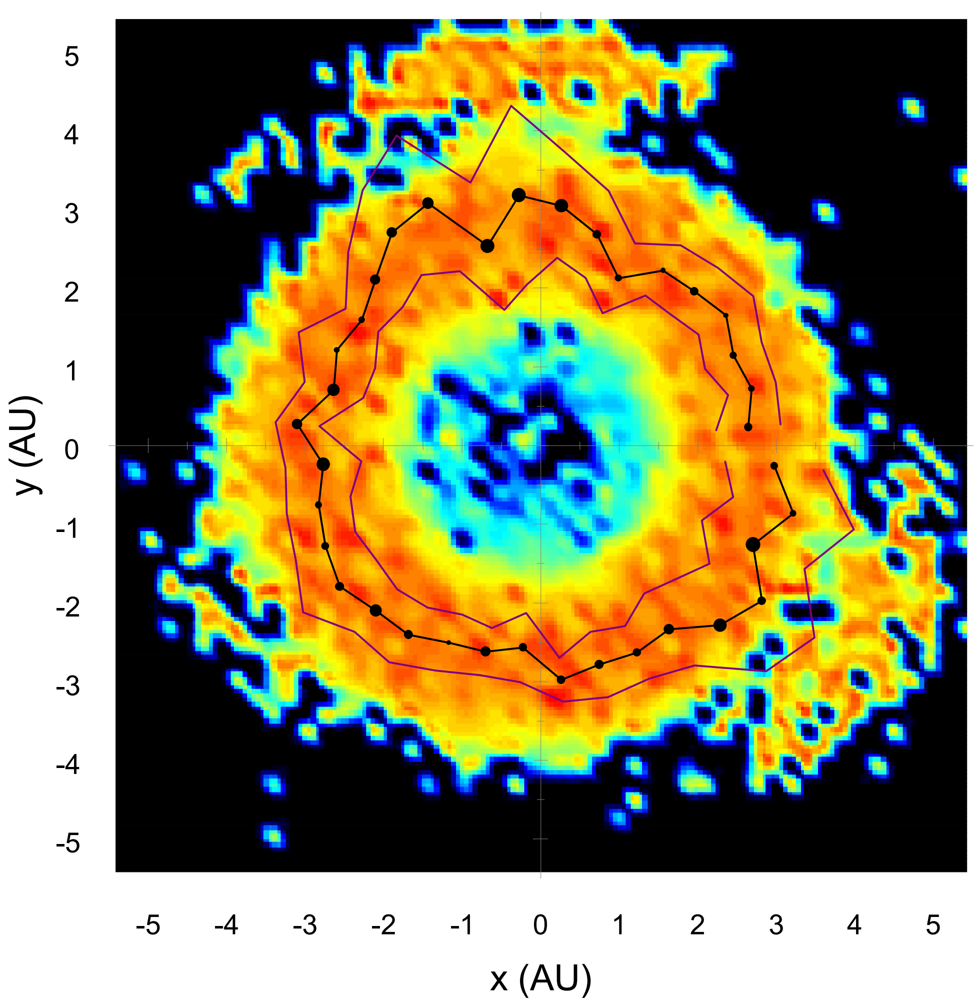
M(θ) = total mass in azimuthal wedge



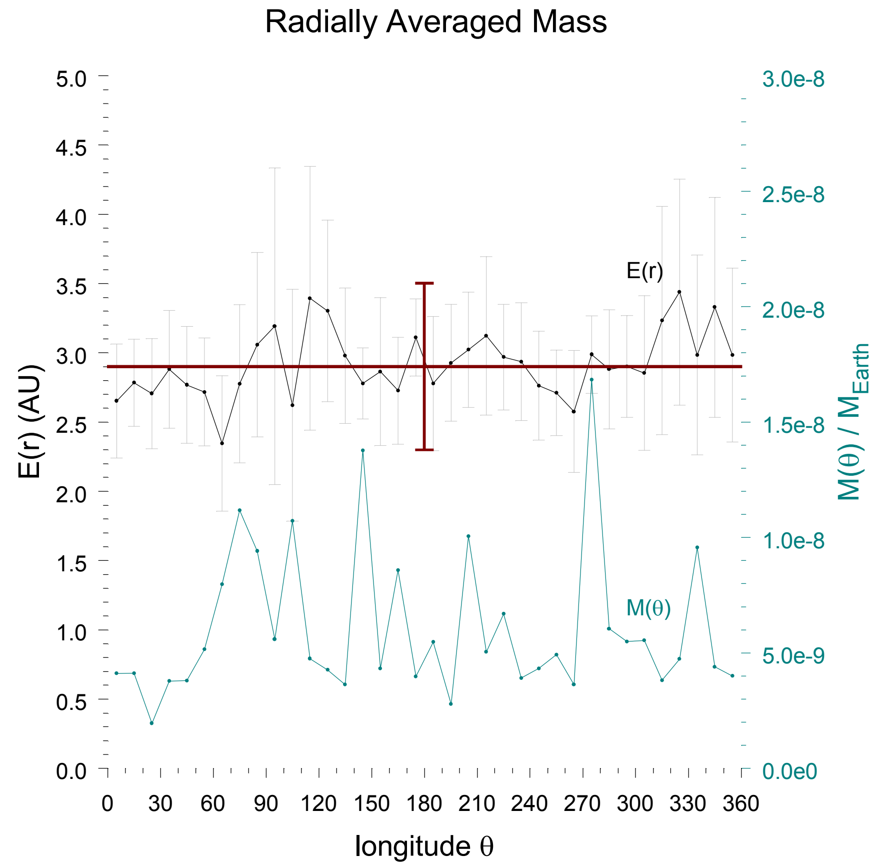
•

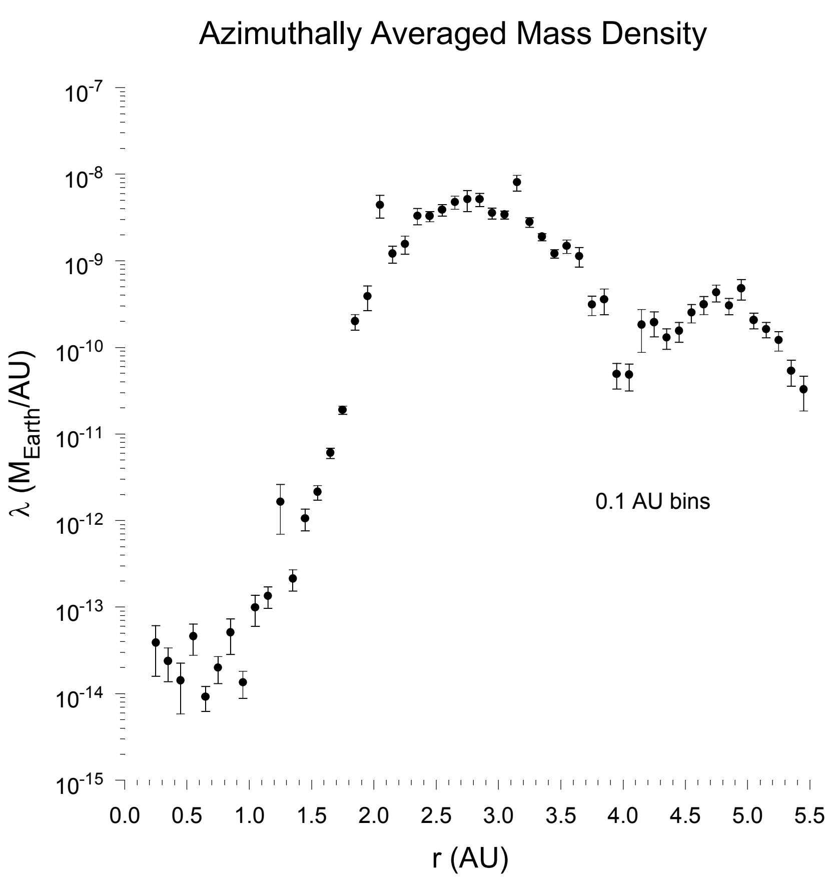
Spatial Mass Distribution: D < 300 km





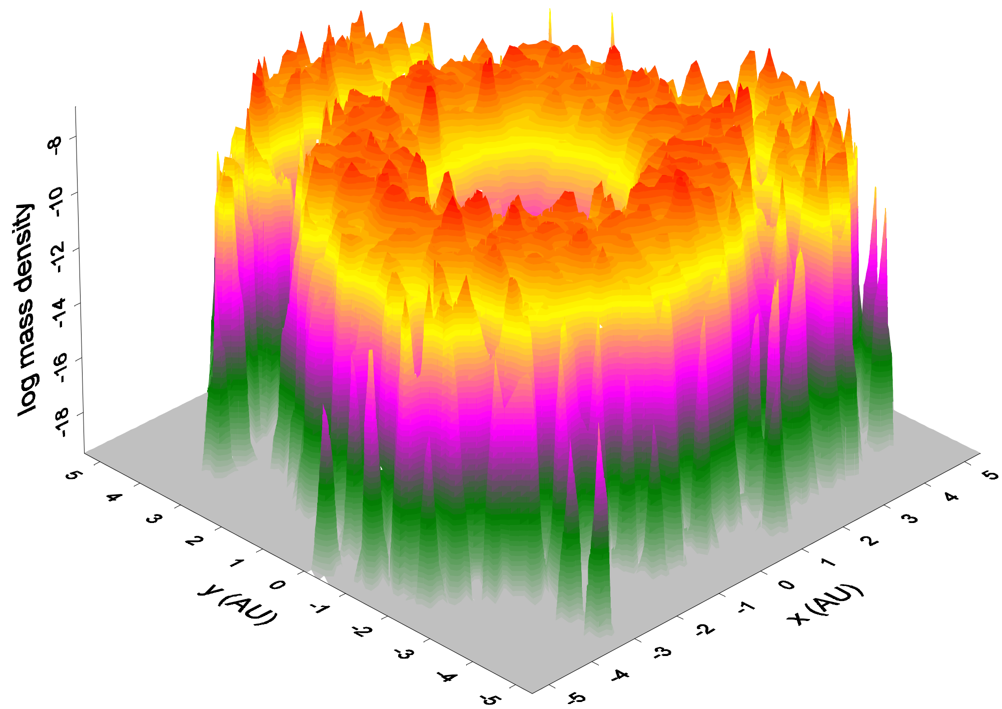
Spatial Mass Distribution: D < 300 km

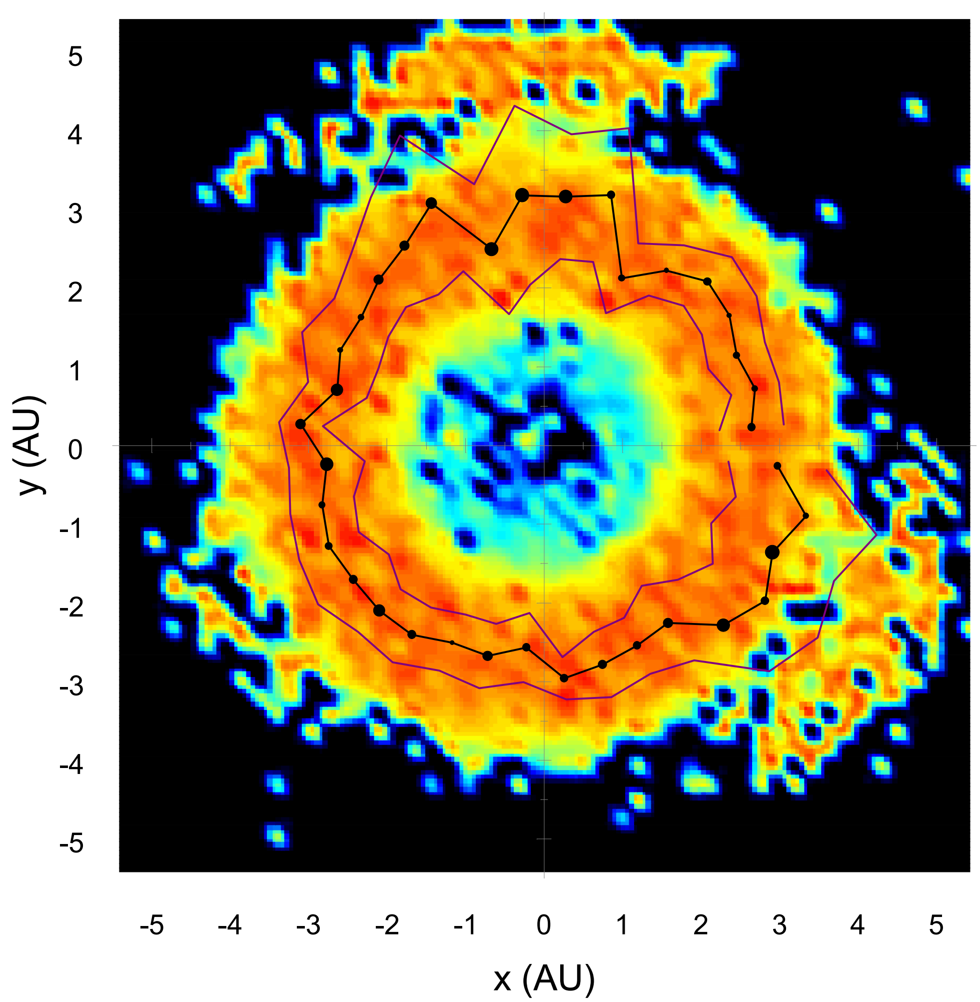




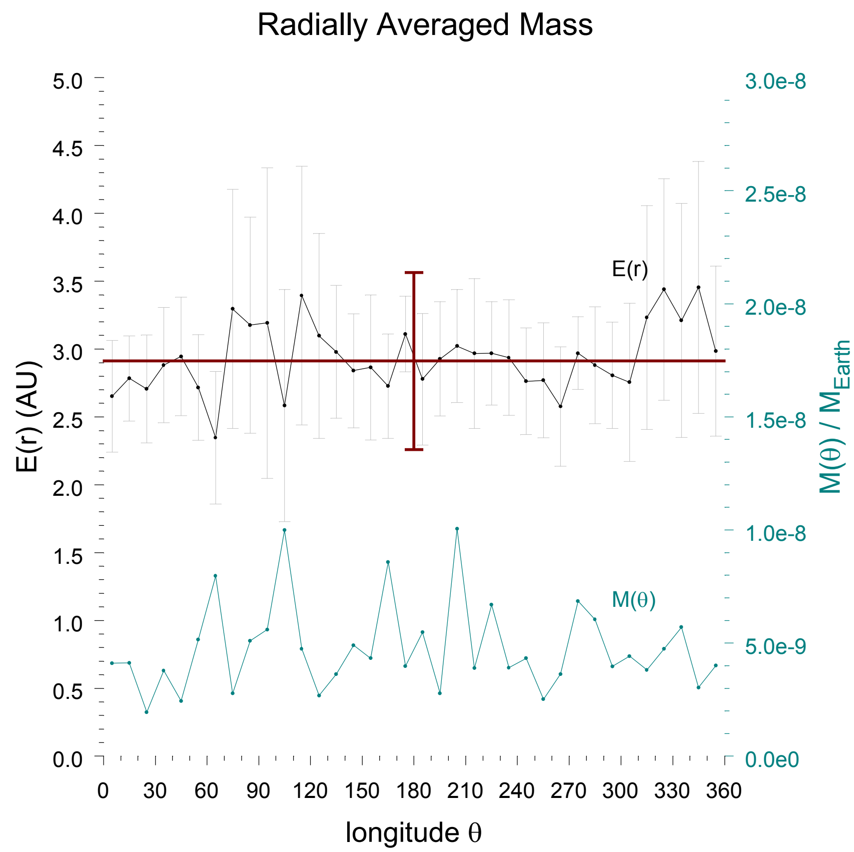
•

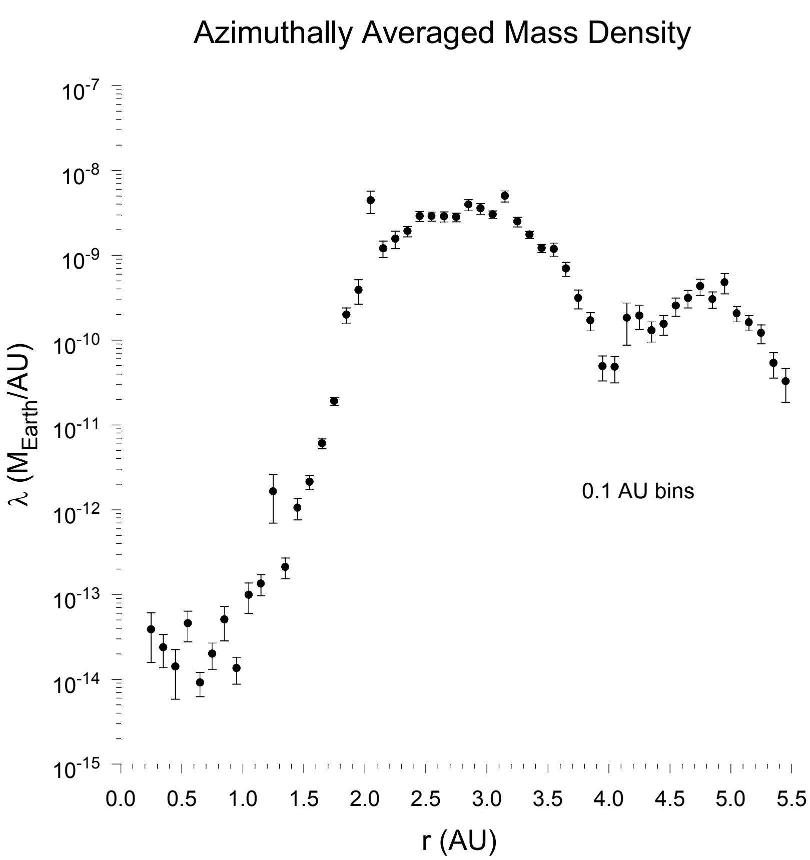
Spatial Mass Distribution: D < 200 km





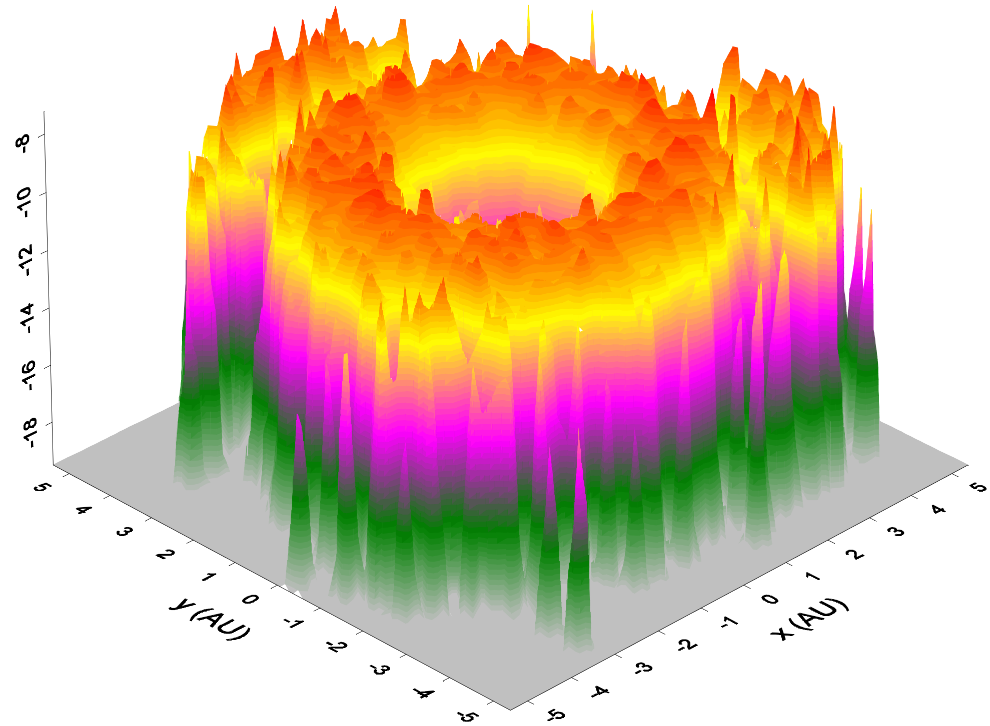
Spatial Mass Distribution: D < 200 km

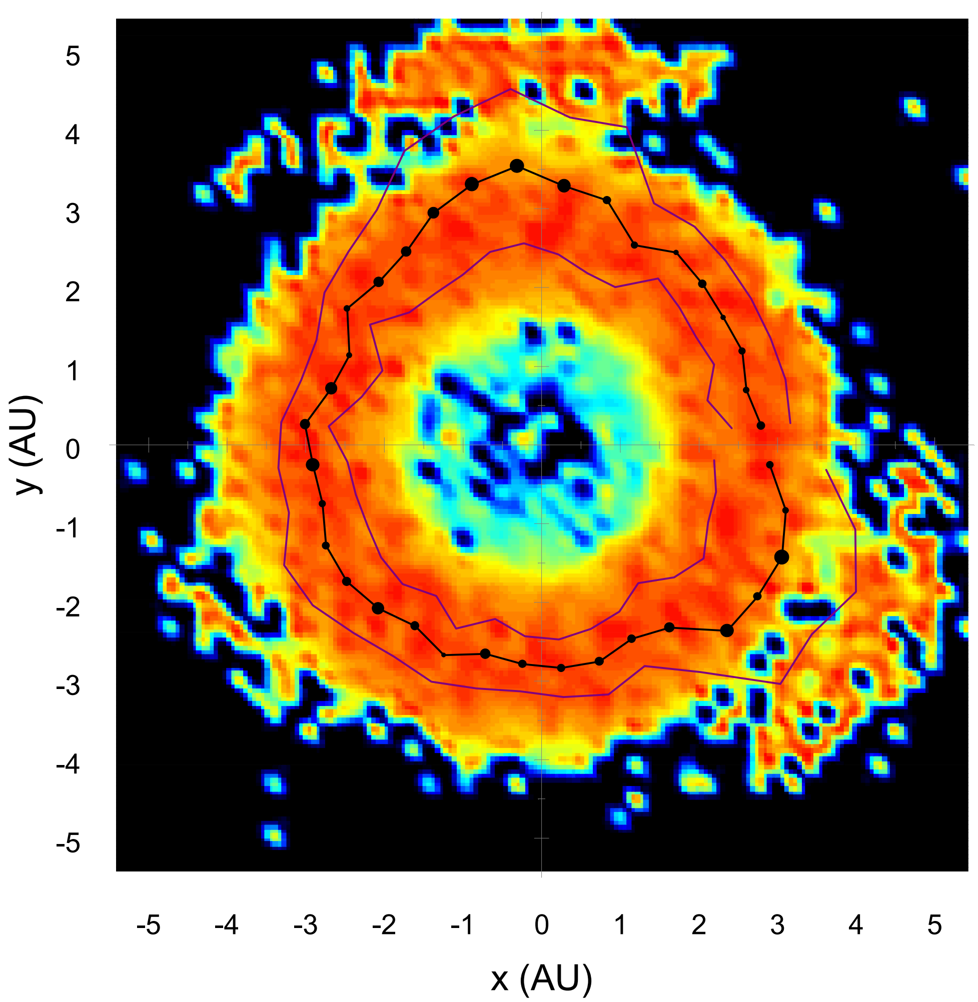




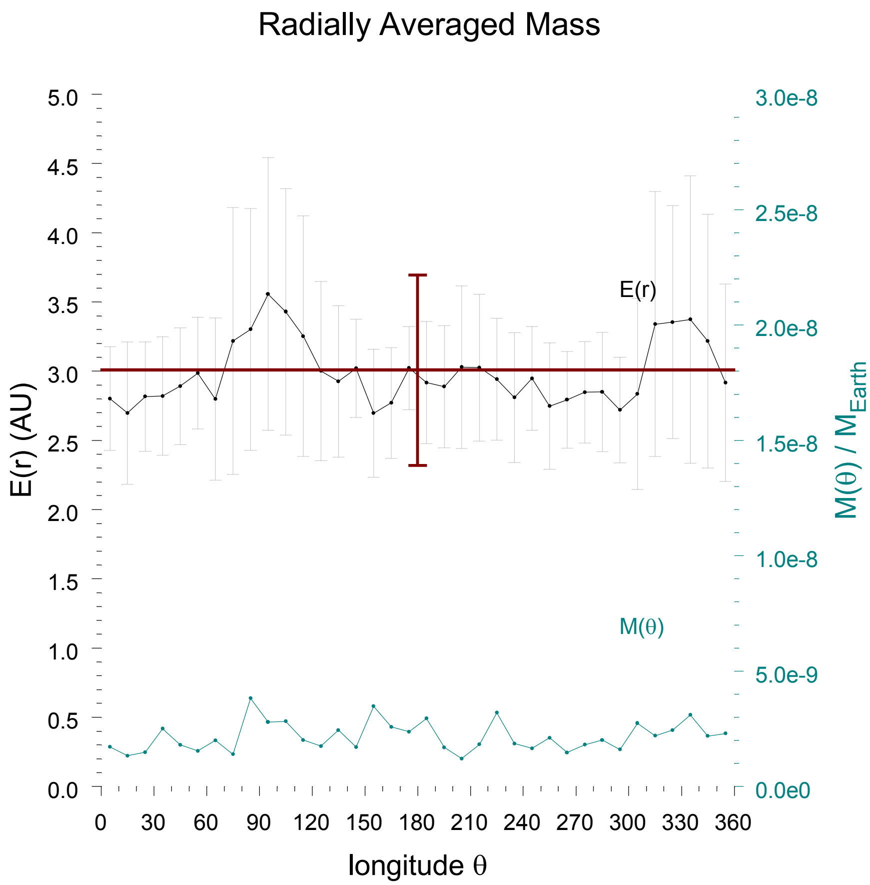
•

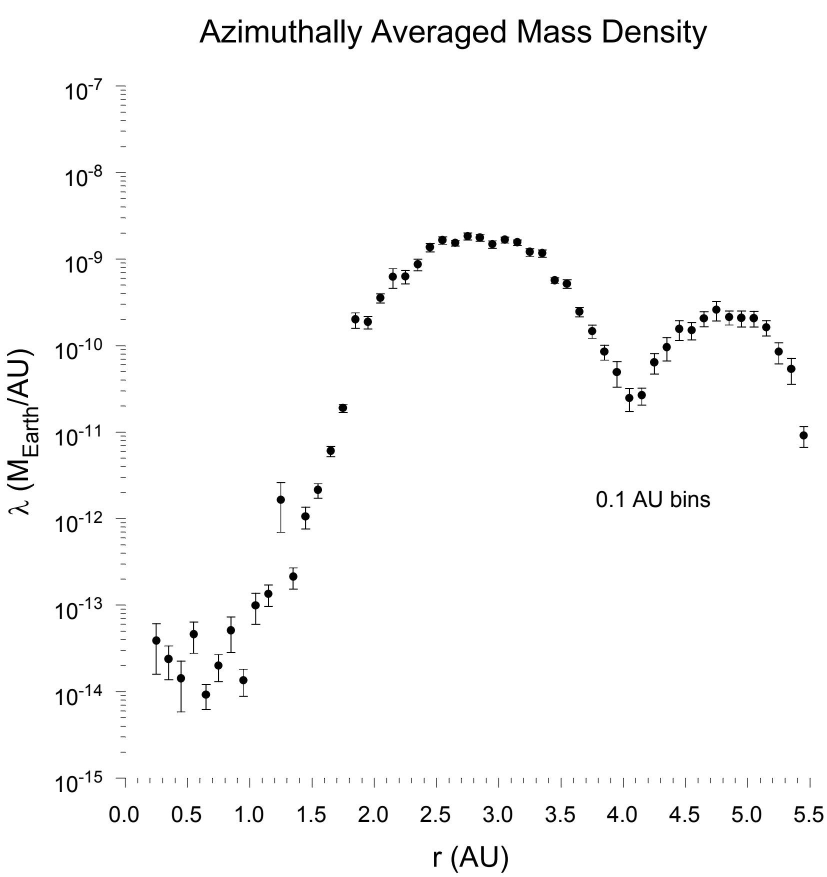
Spatial Mass Distribution: D < 100 km





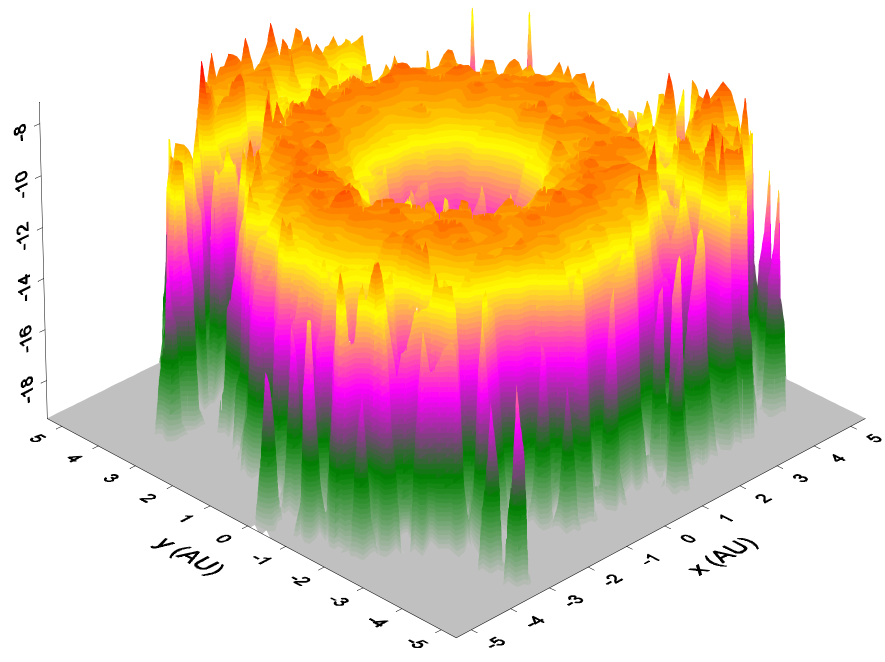
Spatial Mass Distribution: D < 100 km

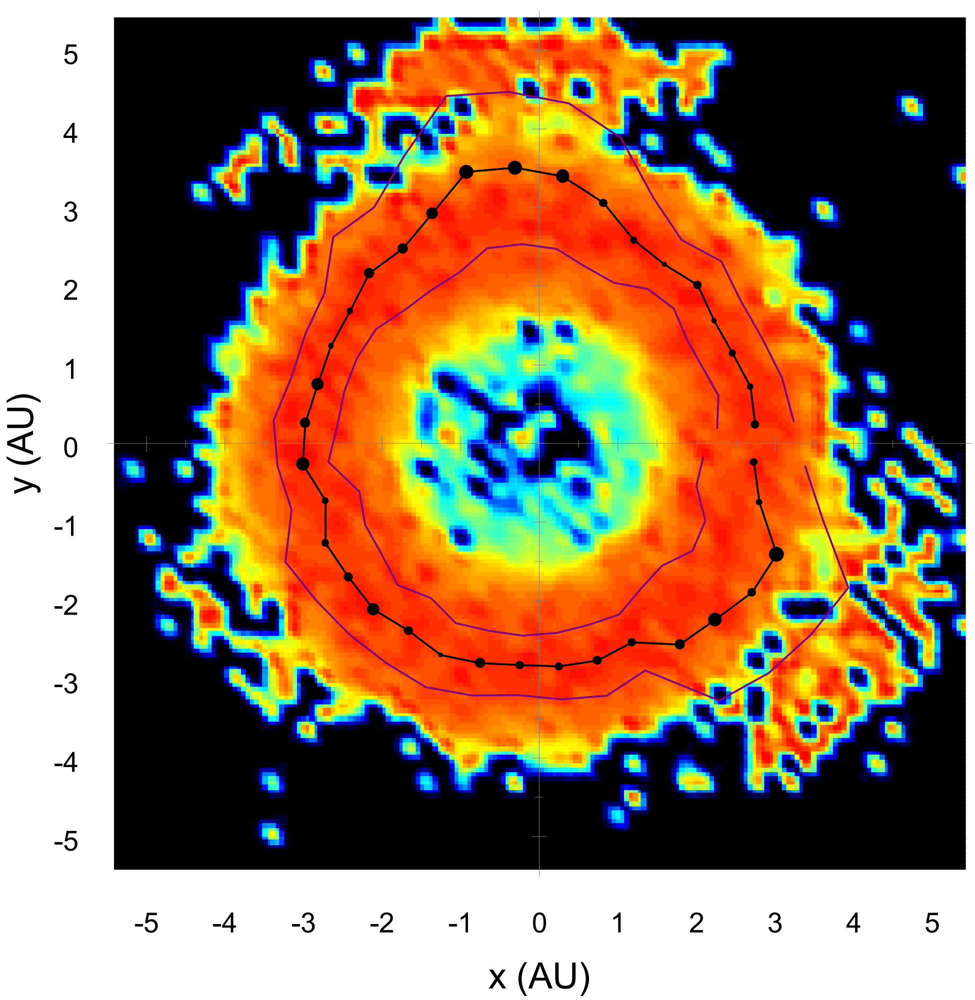




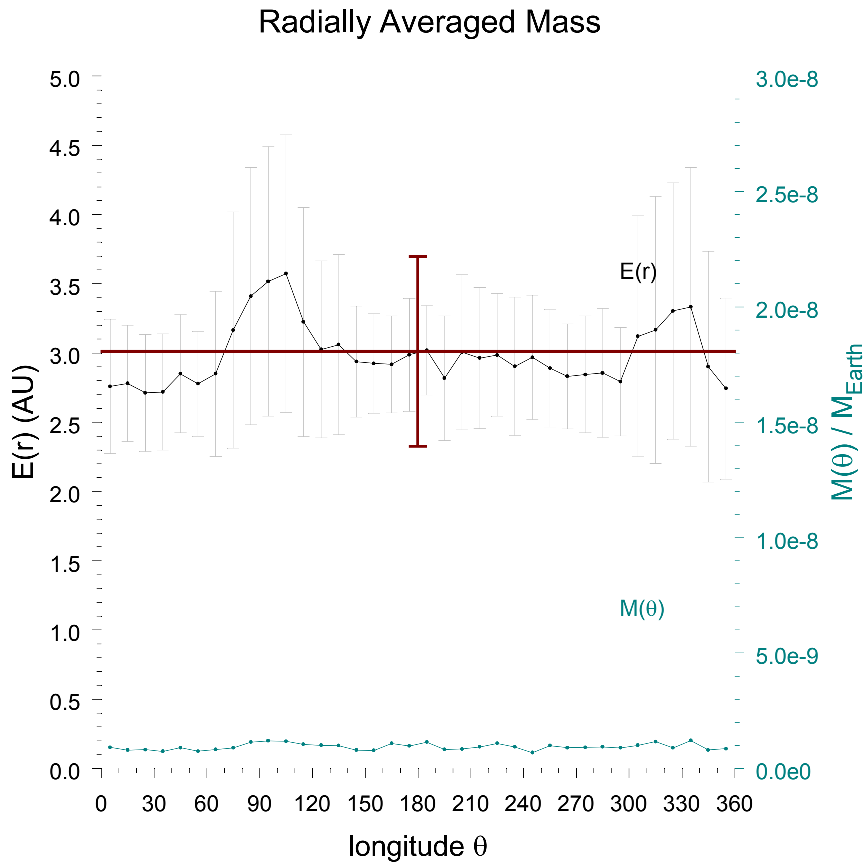
•

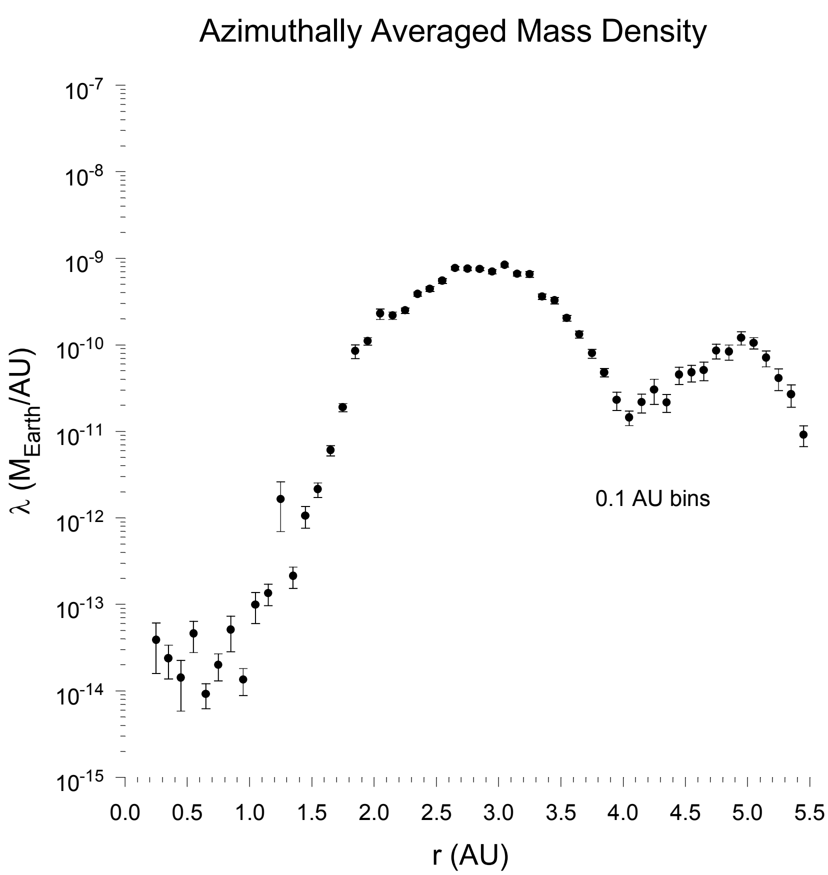
Spatial Mass Distribution: D < 50 km





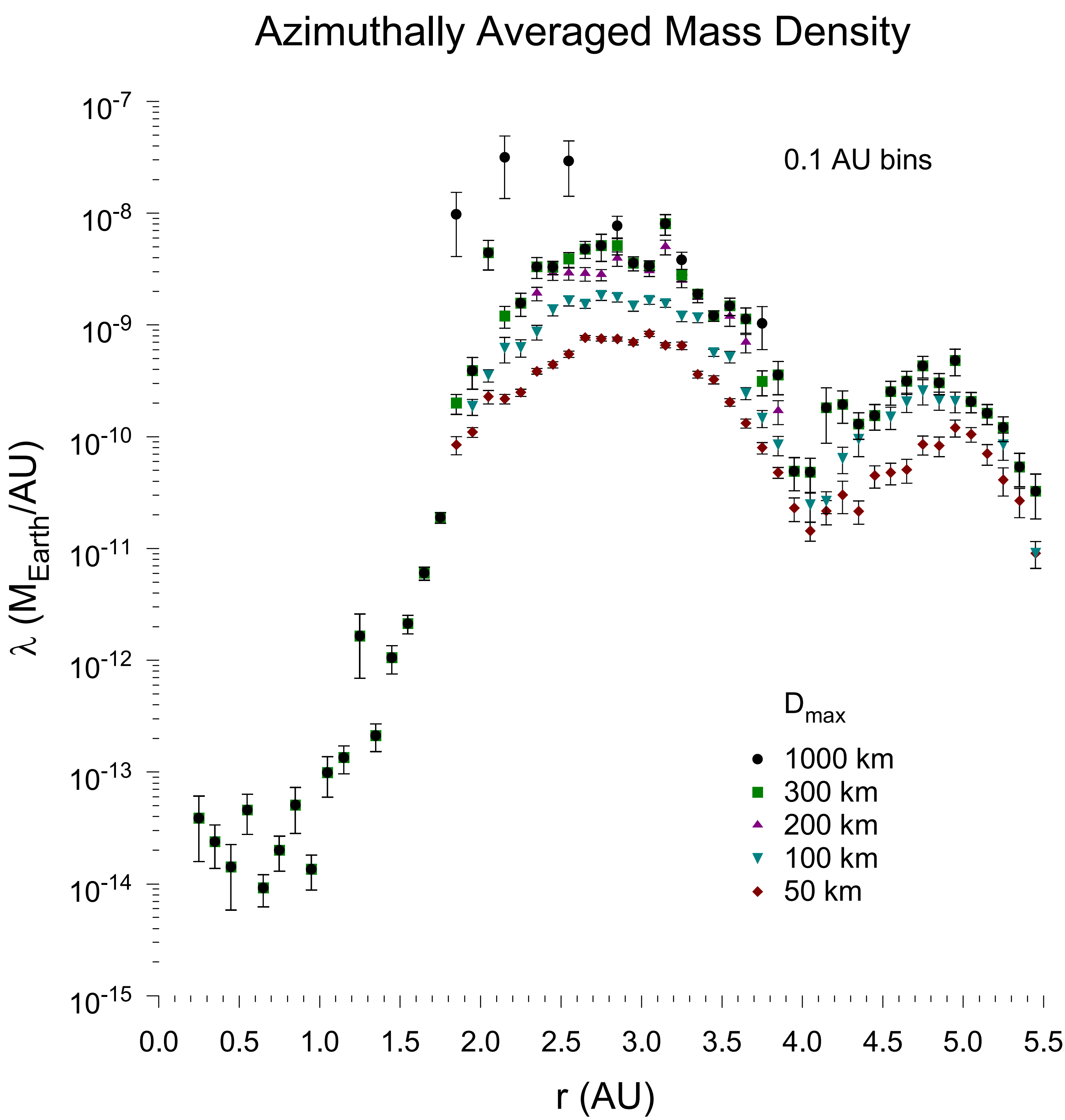
Spatial Mass Distribution: D < 50 km





•

Radial Mass Profile Summary



What to Note about the Spatial Mass Distribution

- Lumpy at all diameter cutoff levels
  - Very lumpy when all asteroids are considered
  - Grows smoother with decreasing diameter cutoff
- Dominated by the Big Three (or 4 or 7 or 11...)
- Jupiter Trojans: large moment arm skews average mass-weighted radial position of a "mean belt" approximation
- What effects might we expect on the inner planets?
  - Large, individual effects from handful of large asteroids
  - Background effects from large number of small asteroids
    - Smooth noise distribution?
  - "Lumpy" effects from intermediate-mass asteroids
    - Effects due to dozens, instead of just a few, individual asteroids
    - Individually too large to just dump into the smooth noise distribution background, but neither are they large enough to dominate

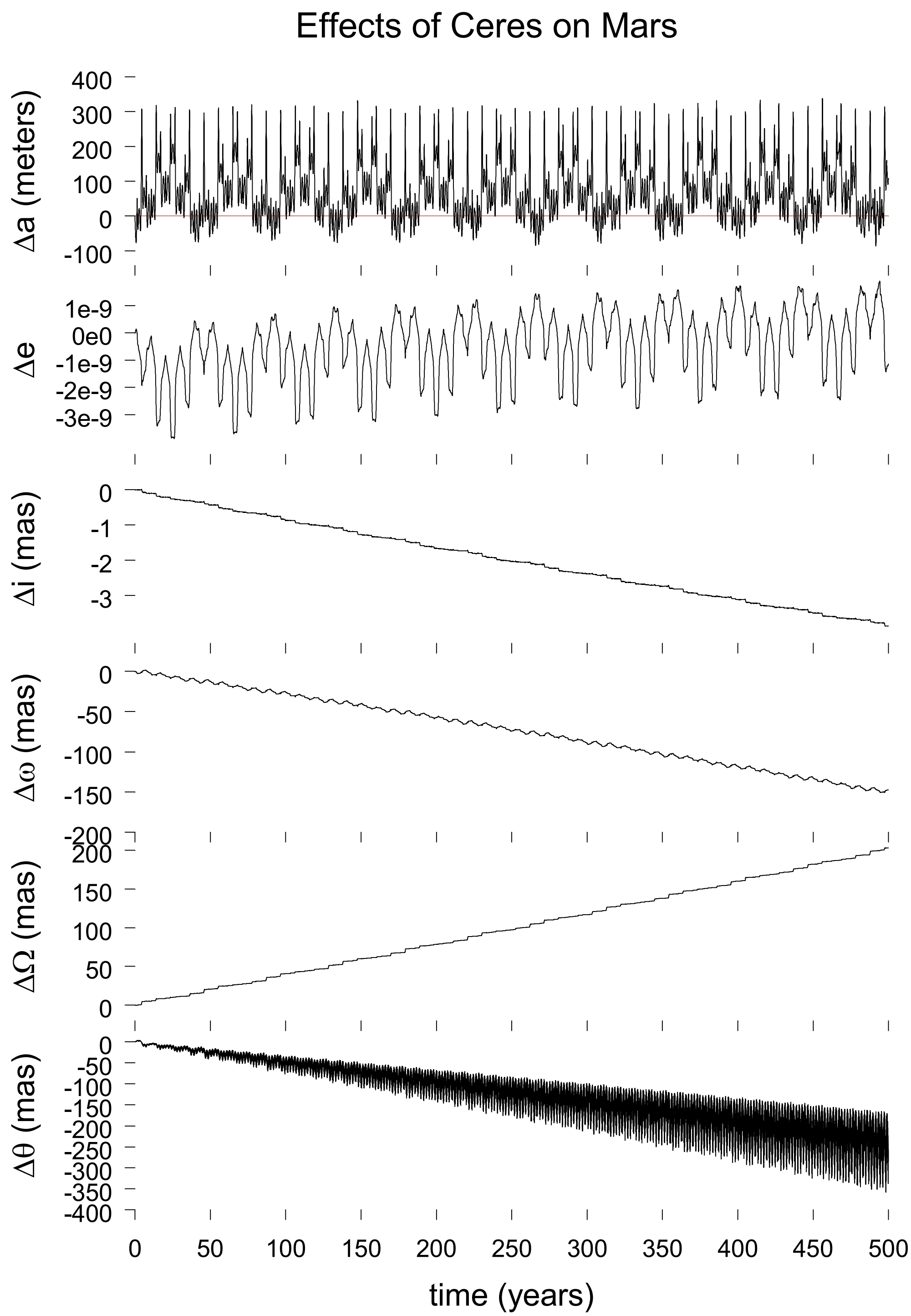
Numerical Integrations for This Preliminary Study

- All major planets
  - i.e., Mercury through Neptune
- All asteroids D > 85 km (310)
  - (as determined from IRAS)
- Newtonian gravity
- Earth+Moon treated as one body
- Integration span: Δt = 500 yr
  - Why 500 years?
    - High-resolution details vs. long-term effects
    - Lyapunov time
      - T_λ ~ 5 Myr — planets
      - T_λ ~ 50-100 yr — NEOs
    - 500 yr seems a reasonable compromise
- Bulirsch-Stoer extrapolation method
  - Symplectic methods inappropriate here
- Measured perturbations

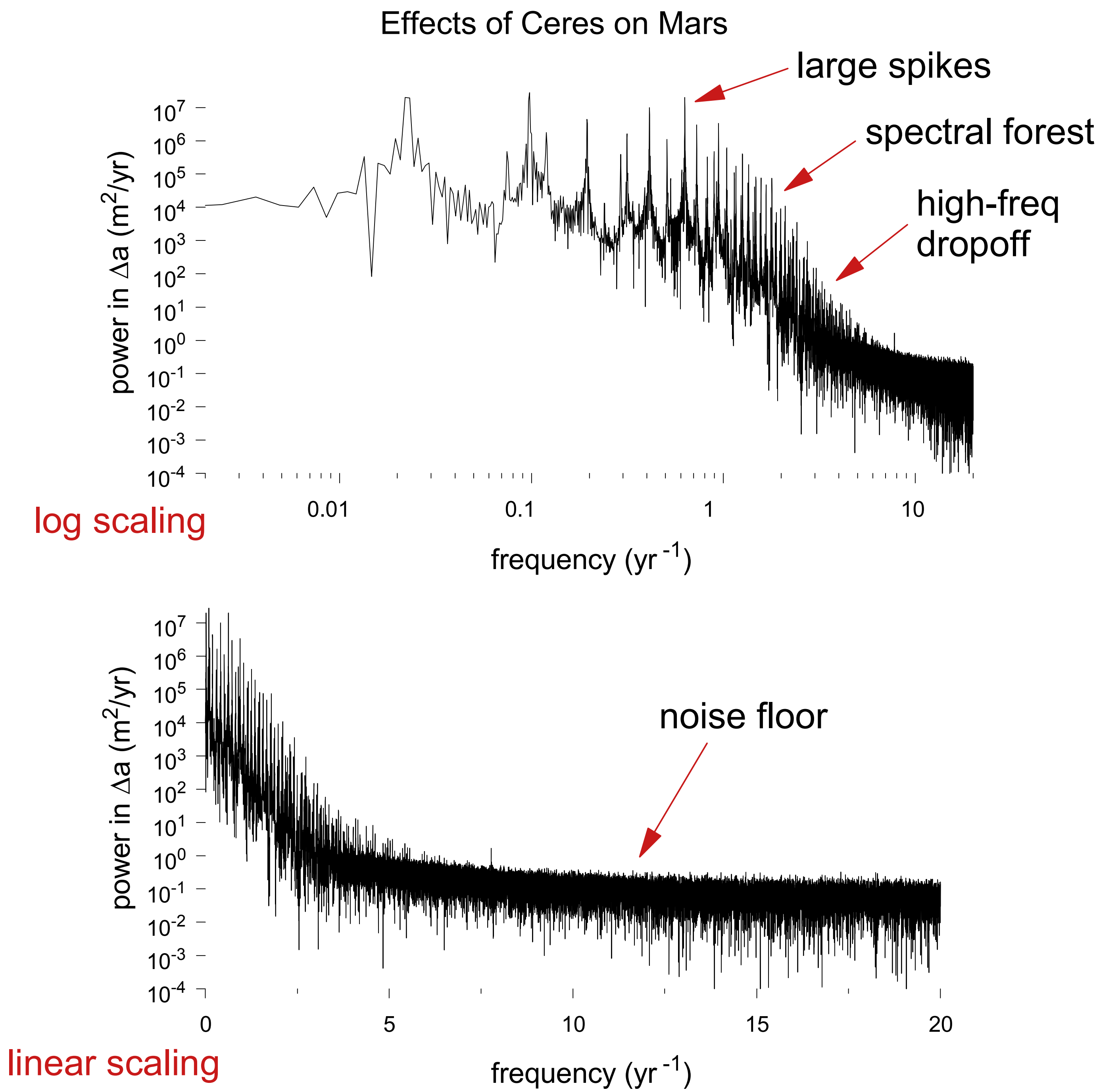
  = I(planets+asteroids) - I(planets)

Ceres Acting on Mars: Orbital Elements

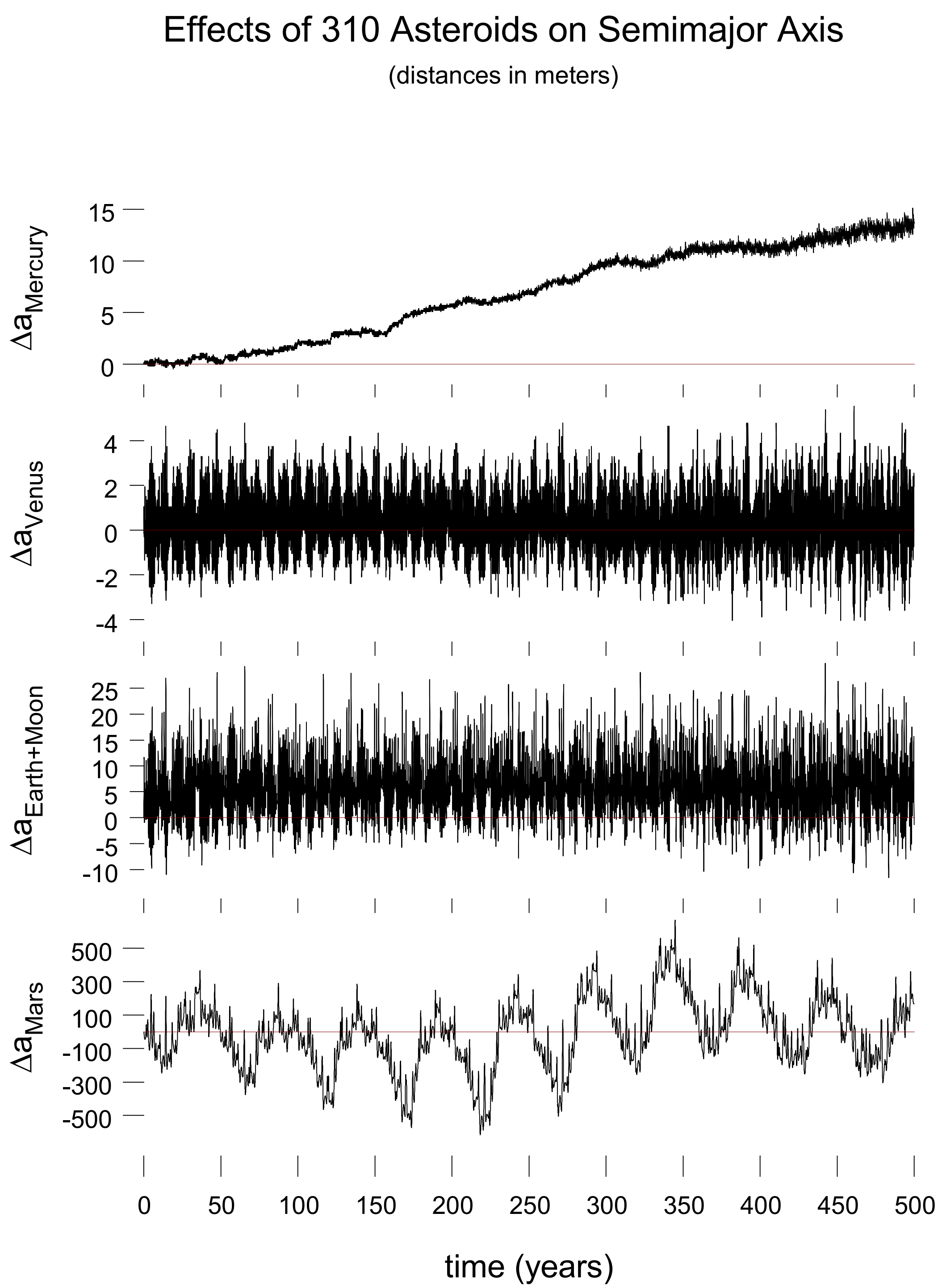
- Mars + planets + only Ceres



Ceres Acting on Mars: Power Spectrum (Δa)

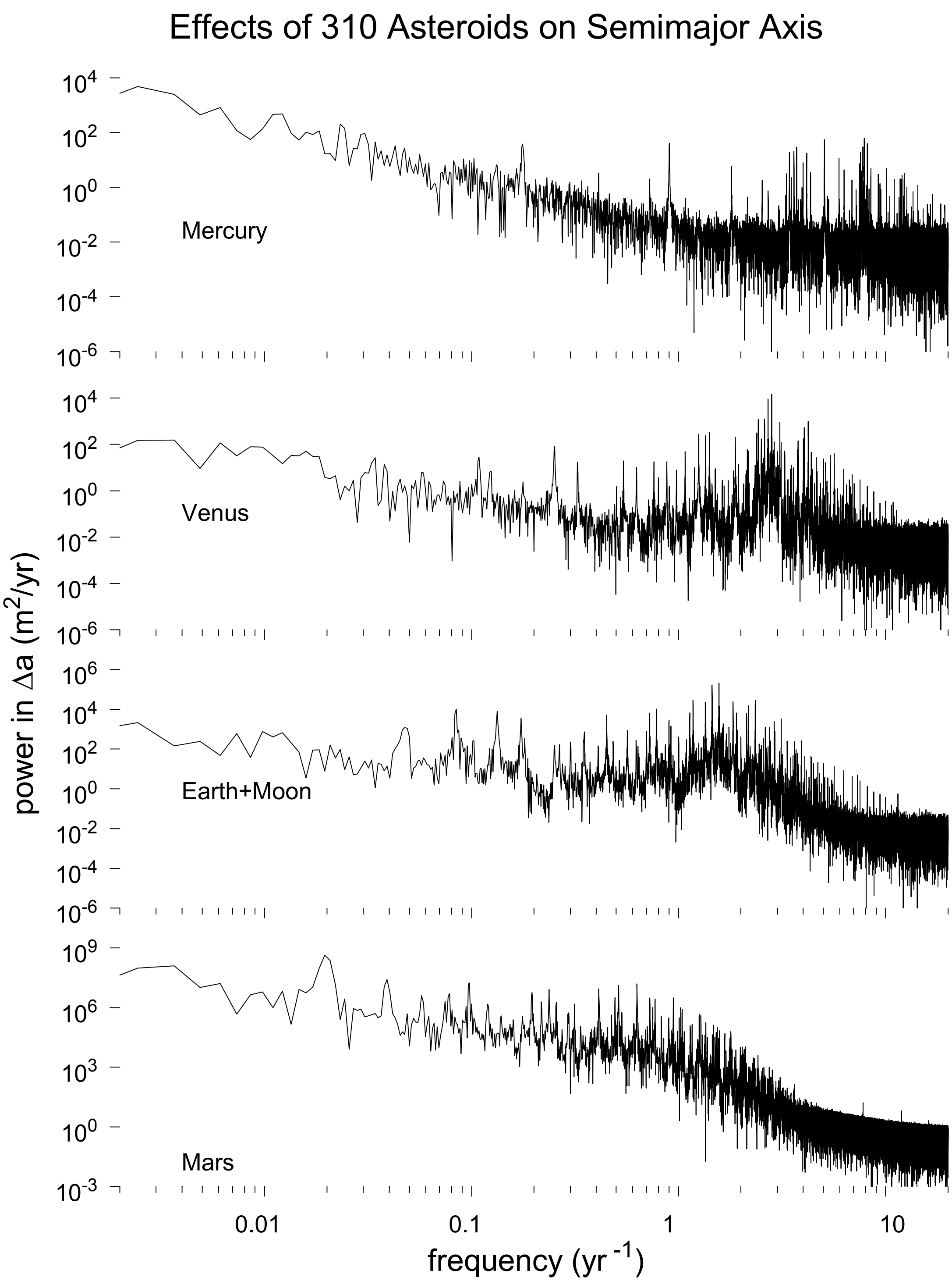


Δa of Planets Due to Asteroid Noise



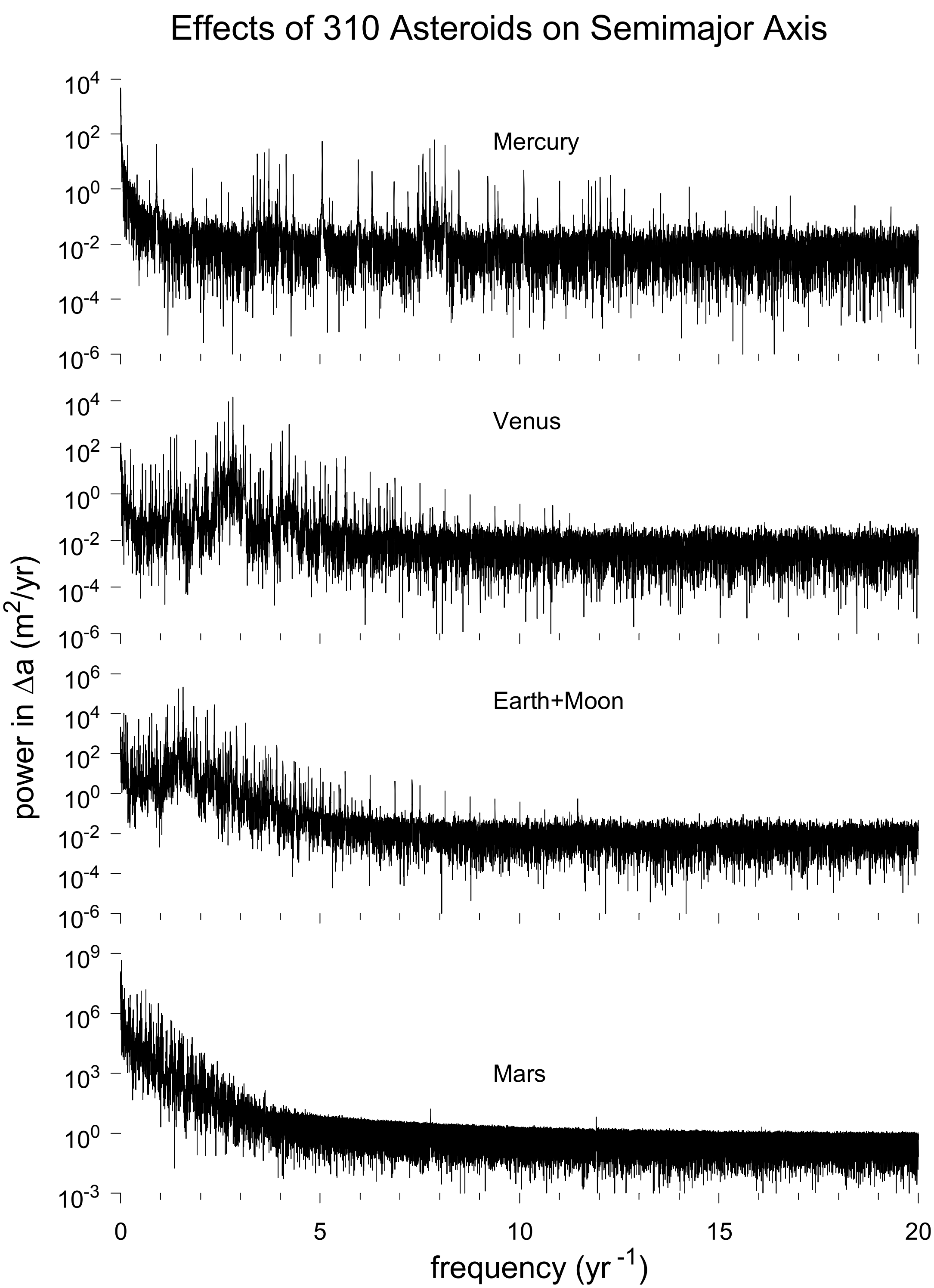
Corresponding Power Spectra

- logarithmic frequency scale (emphasizes low freqs)



Corresponding Power Spectra

- linear frequency scale (emphasizes noise floor)

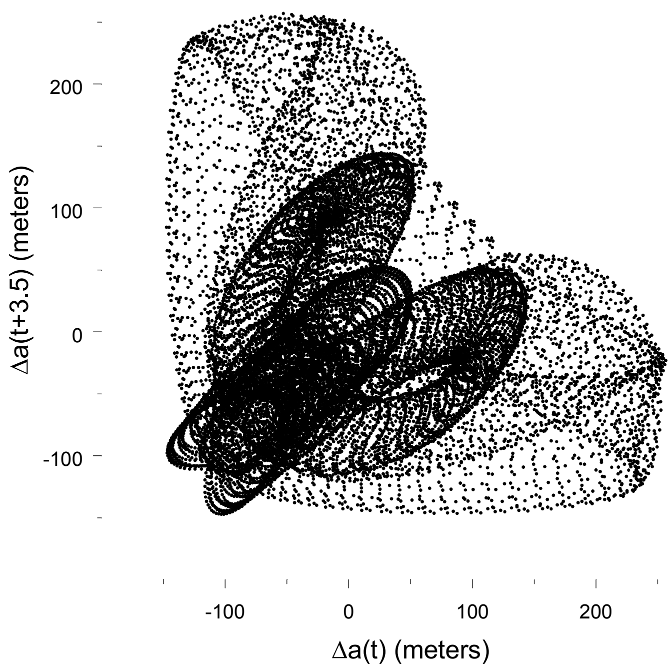


Phase Space Reconstruction from Δa

- Time delay maps can give a qualitative picture of the important dynamics.
- The Takens embedding theorem states that we can determine the geometric structure of the dynamics of a multivariate system from observations of a scalar diagnostic by constructing a time delay map from that scalar.
- In our specific case, choose [$\Delta a(t)$, $\Delta a(t+\Delta t)$]
  - Δt must be large enough for the dynamics to be uncorrelated.
  - Choose $\Delta t > \Delta t_c$, where Δt_c is the first zero crossing of the autocorrelation function for $\Delta a(t)$.

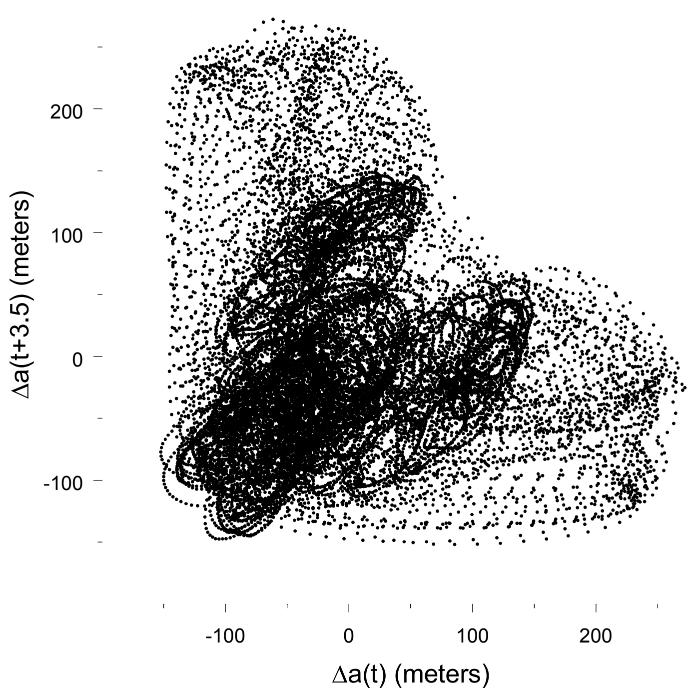
Various Perturbations on Mars: Δa Time Delay

Mars + Ceres



- restricted 3-body problem
- well-defined hypersurface

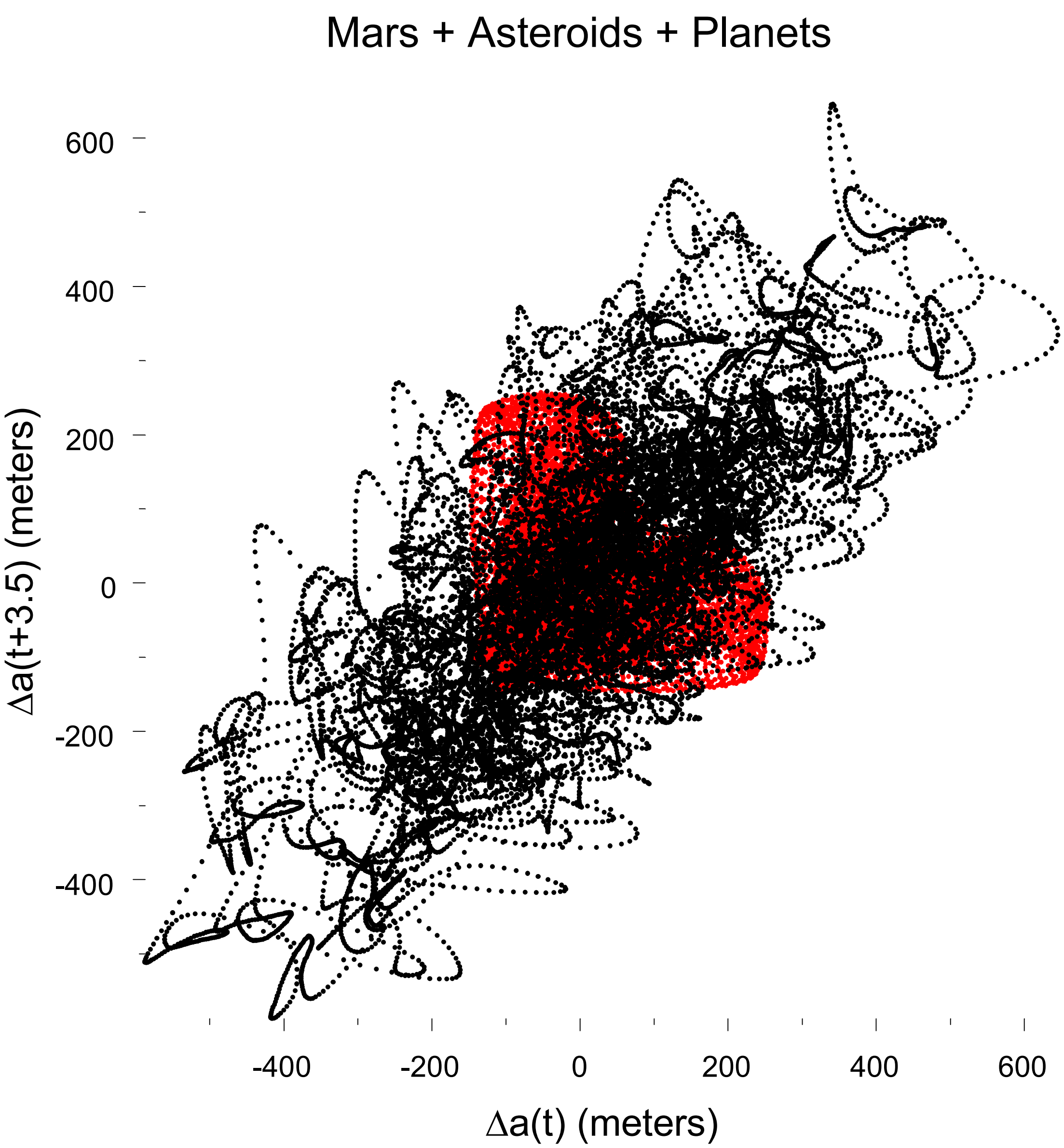
Mars + Ceres + planets



- N=10
- significant disorder
- hypersurface still visible

Various Perturbations on Mars: Δa Time Delay

- N=319
- Hypersurface completely destroyed



Preliminary Results Summary

- Mass distribution
  - Dominated by largest few asteroids
  - lumpy!

planet	Δa (m)	Δnoise (O)
Mercury	<1	—
Venus	a few	<1/2
Earth+Moon	a few tens	~1
Mars	several hundred	~2

- Power spectra:
  - Large low-frequency broadband power
  - Forest of narrow-band spikes
  - Ramp down (several orders of magnitude) to noise floor
  - Venus, Earth (Mars? Mercury?):
    - Broad spectral feature
    - Indicative of chaotic motion

Preliminary Results Summary (continued)

- Disorder of phase space (Mars)
  - Mars+Ceres alone: low-dimensional hypersurface is well defined
  - Add the planets: minor perturbations, but qualitative features of hypersurface still visible
  - Add the rest of the asteroids: very large effect, and the hypersurface is utterly destroyed
    - Indicative of chaotic motion
- Low-order mean-motion resonances: currently, important only for Mars
- ...to be continued
  - Interpretation of power spectra