With every passing hour our solar system comes forty-three thousand miles closer to globular cluster M13 in the constellation Hercules, and still there are some misfits who continue to insist that there is no such thing as progress.

-- Ransom K. Ferm

Distributions of Observation Density on the Sky and Scan Angle

(Current Status)

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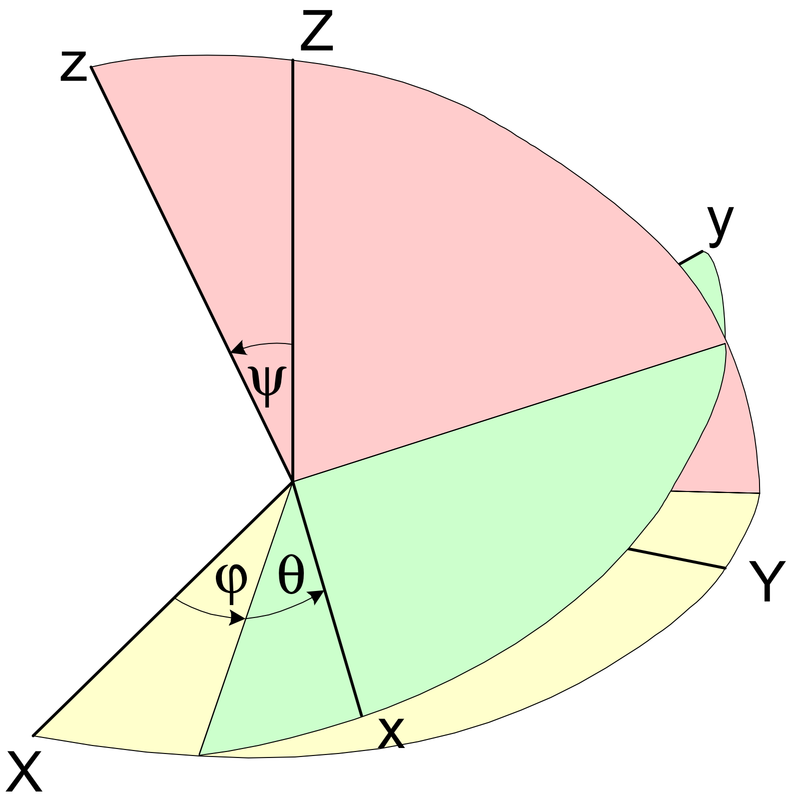
See
http://aa.usno.navy.mil/murison/FAME/ObservationDensity/
for latest results

Introduction

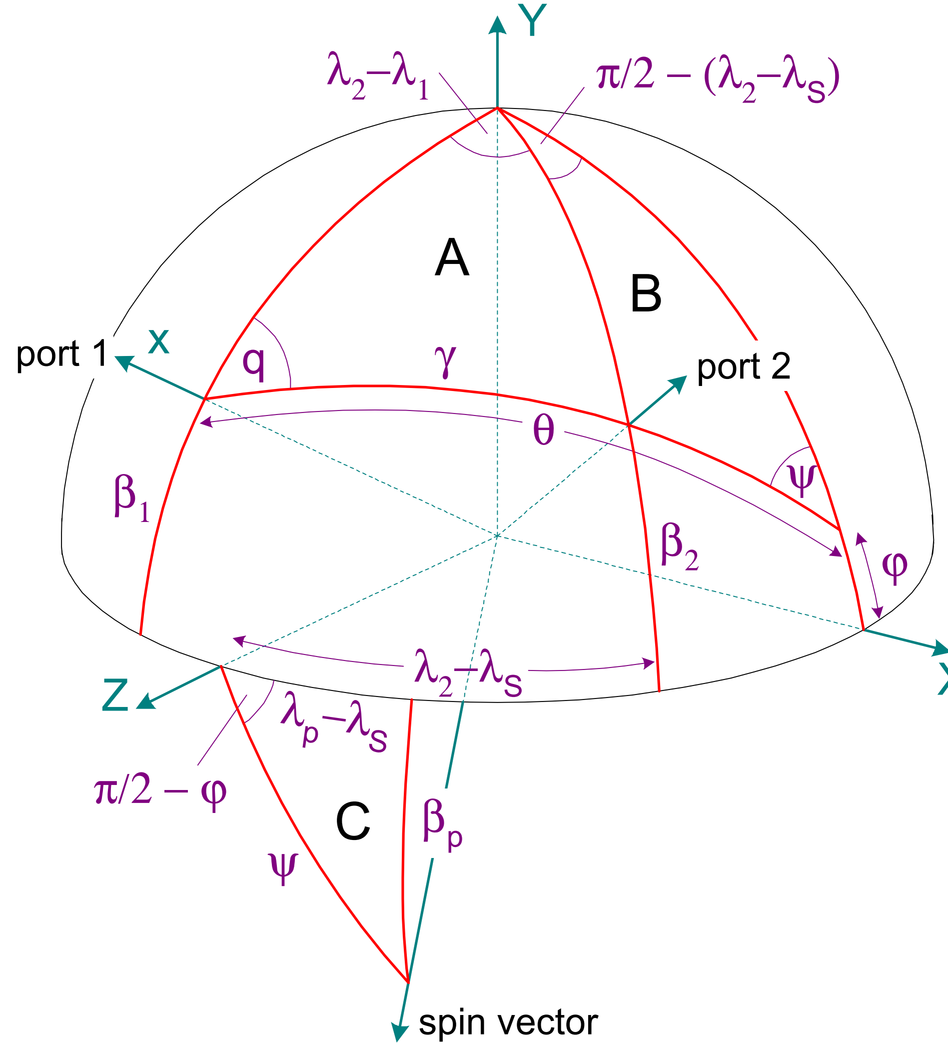
- Goal: minimize mission-averaged astrometric errors from an observation density and uniformity perspective
- Two distributions fundamentally affect the mission astrometric errors
  - Distribution of observations on the sky
  - Distribution of scan angle
    - angle between instantaneous scan direction and a meridian through the ecliptic north pole
    - determines orientation and ratio of error ellipse axes
- Ideally, both of these distributions should be
  - dense
  - homogeneous
- Intuitively, the distribution homogeneity depends on
  - Length of mission
  - Sun angle
  - Precession rate
- The Problem: What are the effects on mission-averaged astrometric accuracies of changing
  - the Sun angle
  - the precession rate

Geometry

- Body frame [x,y,z] and "external" frame [X,Y,Z] linkage via Euler angles



- Spherical geometry



Geometry *(continued)*

- Geometry equations describing behavior: not hideous, but not trivial, either

$$\cos\lambda = \frac{\cos\lambda_S \, \sin\theta\sin\psi + (-\cos\theta\cos\phi + \cos\psi\sin\theta\sin\phi)\sin\lambda_S}{\sqrt{1-(\sin\phi\cos\theta + \cos\phi\sin\theta\cos\psi)^2}}$$

$$\sin\lambda = \frac{\sin\theta\sin\psi\sin\lambda_S - \cos\lambda_S\,(-\cos\theta\cos\phi + \cos\psi\sin\theta\sin\phi)}{\sqrt{1-(\sin\phi\cos\theta + \cos\phi\sin\theta\cos\psi)^2}}$$

$$\sin q = Q \qquad \cos q = -\frac{-\cos^2(\lambda-\lambda_S)\sin^2\beta + \sin^2(\lambda-\lambda_S)\cos\psi}{\sin(\lambda-\lambda_S)[1-\cos^2(\lambda-\lambda_S)\cos^2\beta]} - \frac{\cos(\lambda-\lambda_S)\sin\beta(\cos^2\psi - \sin^2(\lambda-\lambda_S))}{\sin(\lambda-\lambda_S)[1-\cos^2(\lambda-\lambda_S)\cos^2\beta]\,Q}$$

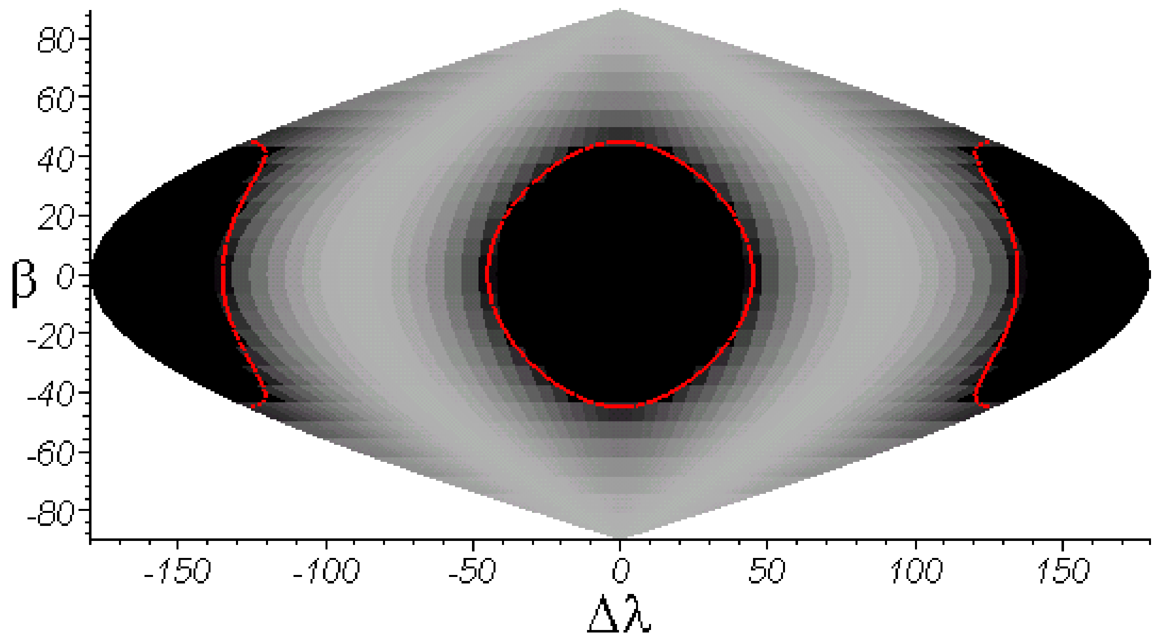
$$Q = \frac{\cos(\lambda-\lambda_S)\cos\psi\sin\beta \pm |\sin(\lambda-\lambda_S)|\sqrt{-\cos^2(\lambda-\lambda_S)\cos^2\beta + \sin^2\psi}}{1-\cos^2(\lambda-\lambda_S)\cos^2\beta}$$

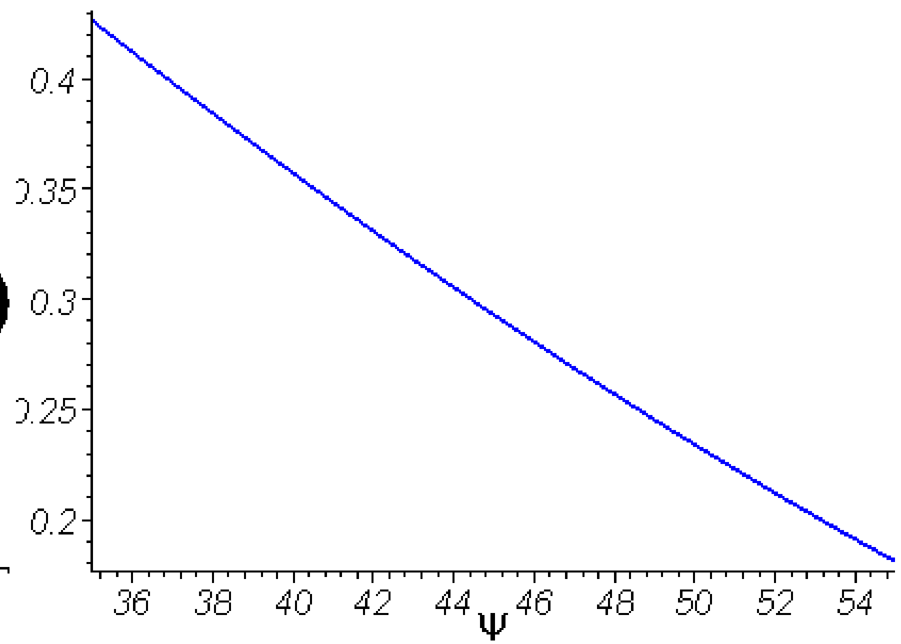
$$\cos\phi = \frac{Q\cos\beta}{\sin\psi} \qquad \sin\phi = -\frac{Q\sin\beta}{-\sin(\lambda-\lambda_S)\sin\psi} + \frac{\cos\psi\cos(\lambda-\lambda_S)}{-\sin(\lambda-\lambda_S)\sin\psi}$$

$$\cos\theta = \frac{[\sin^2\psi\sin\beta - Q\cos^2\beta\cos(\lambda-\lambda_S)\cos\psi]\sin(\lambda-\lambda_S)}{\sin\psi[Q\sin\beta - \cos\psi\cos(\lambda-\lambda_S)]} \qquad \sin\theta = \frac{\cos(\lambda-\lambda_S)\cos\beta}{\sin(\psi)}$$

Geometry *(continued)*

- Precession cone "holes" in Sun and anti-Sun directions
  - hole boundaries determined by sqrt term of Q going imaginary
  - hole angular radius = 90 deg - Sun angle
  - immediate result: larger Sun angle is better

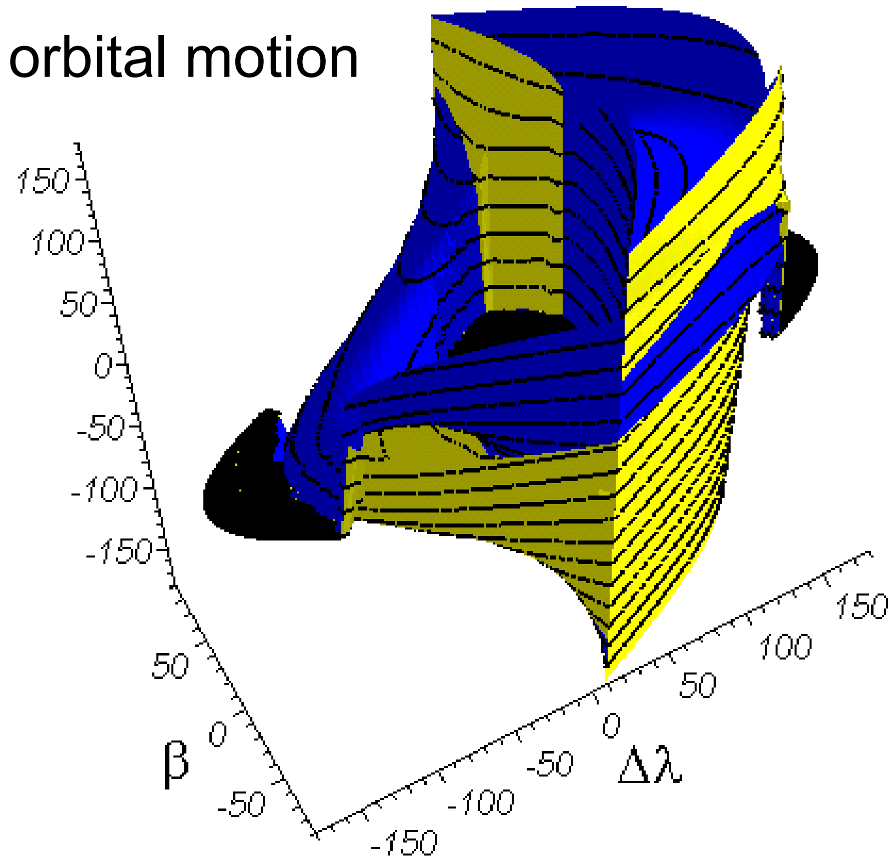




fraction of sky covered by precession cone holes

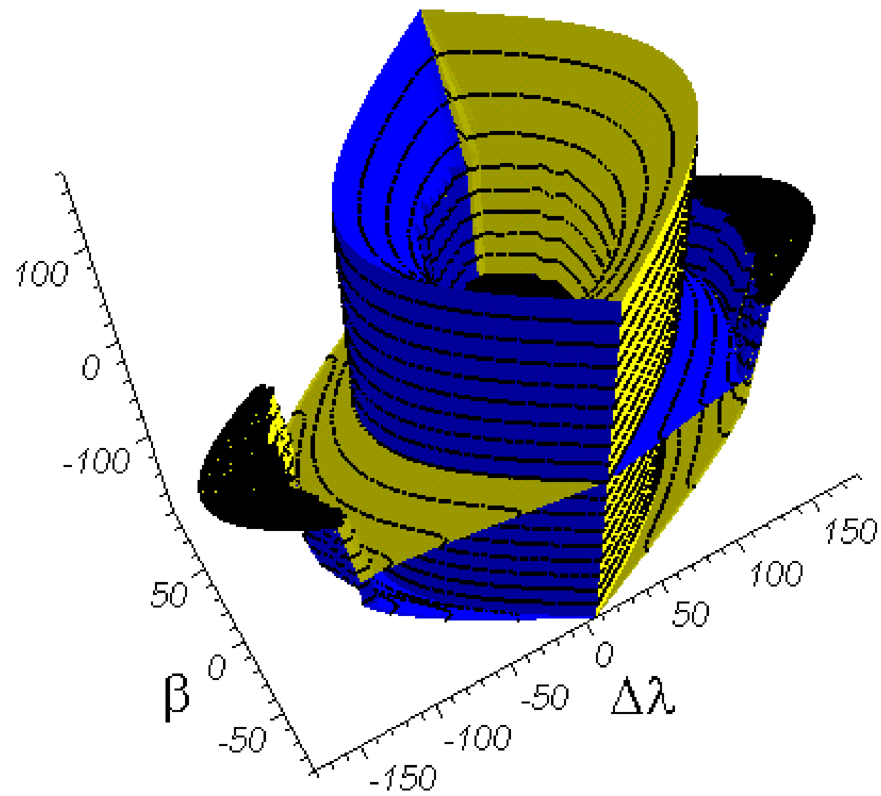
Geometry *(continued)*

- Scan angle q as a function of position on the sky
  - longitude coordinate is wrt Sun's ecliptic longitude
- Two solution surfaces
  - due to quadratic solution pair Q
  - smoothly join at discontinuities
- Will smear in longitude due to Earth's orbital motion



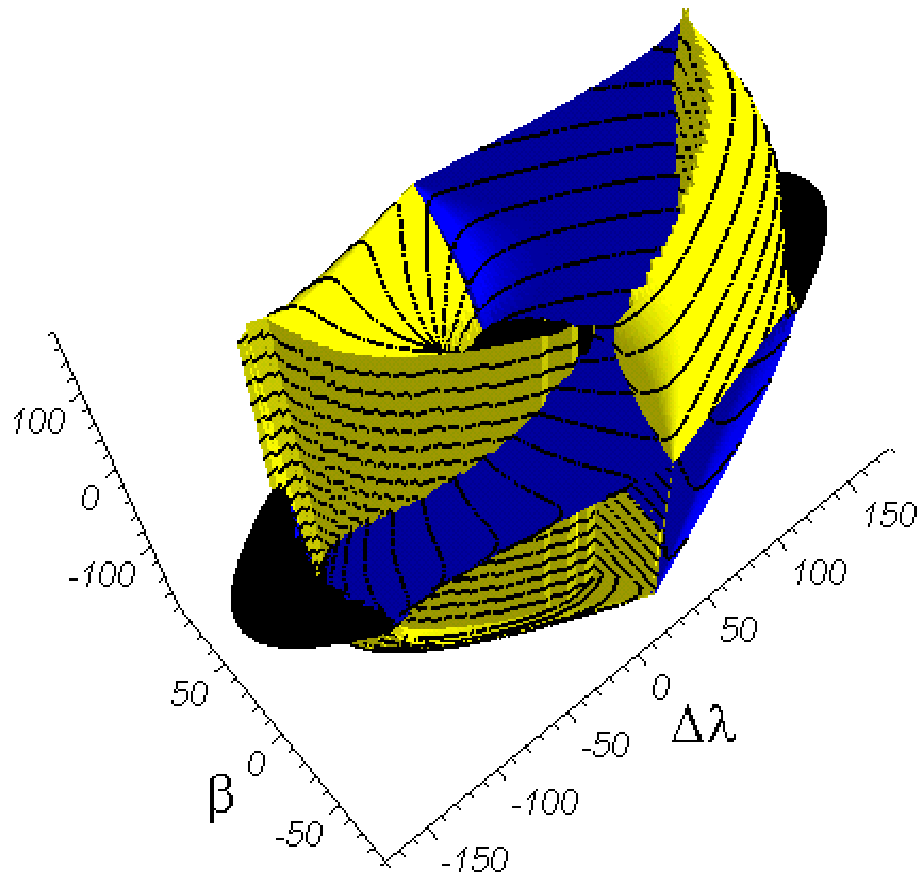
Geometry *(continued)*

- Fast Euler angle θ as a function of position on the sky
  - longitude coordinate is wrt Sun's ecliptic longitude
- Similarly, two solution surfaces due to Q



Geometry *(continued)*

- Slow Euler angle φ (the precession phase angle) as a function of position on the sky
  - longitude coordinate is wrt Sun's ecliptic longitude
- Again, two solution surfaces due to Q

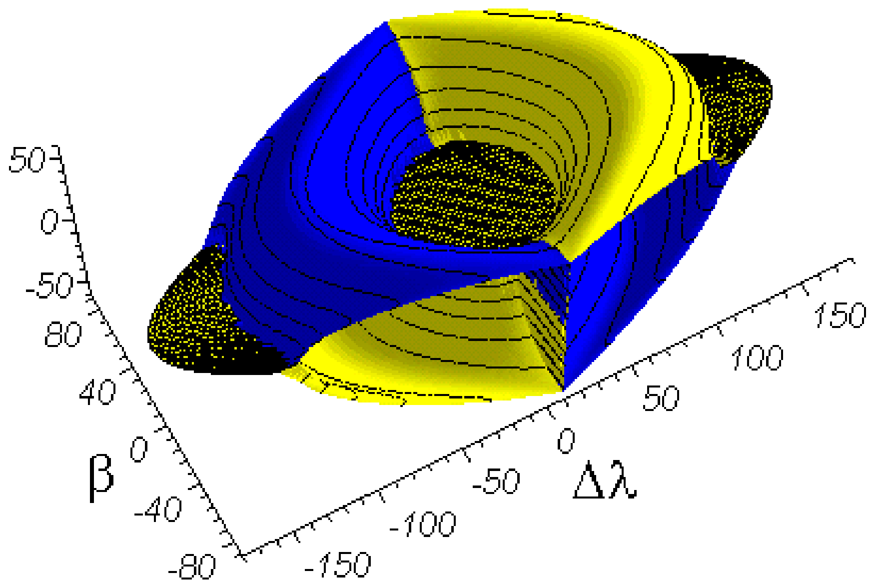


Rotation Rates

- Three orthogonal rotations
  - field rotation
  - cross-scan
  - in-scan $\qquad \Omega_s = \frac{d\theta}{dt} + \frac{d\phi}{dt}\cos\psi - Q\cos\beta\frac{d\lambda_S}{dt}$
- Decompose total angular velocity vector along these three rotation axes (which happen to correspond to body-frame [x,y,z], respectively)
- Cross-scan rate
  - determines density of observations on the sky
  - dominated by precession signal
- $\Omega_r = \cos(\lambda-\lambda_S)\cos\beta\frac{d\phi}{dt} + \frac{\left[\sin^2\psi\sin\beta - Q\cos^2\beta\cos(\lambda-\lambda_S)\cos\psi\right]\sin(\lambda-\lambda_S)}{\sin\psi\left[Q\sin\beta-\cos\psi\cos(\lambda-\lambda_S)\right]}\frac{d\psi}{dt} + \sin\beta\frac{d\lambda_S}{dt}$
- $$\begin{aligned}\Omega_c \;=\; & -\frac{\cos(\lambda-\lambda_S)\cos\beta}{\sin\psi}\frac{d}{dt}\psi \\ & -\frac{[\sin^2\psi\sin\beta - Q\cos^2\beta\cos(\lambda-\lambda_S)\cos\psi]\sin(-\lambda+\lambda_S)}{Q\sin\beta-\cos\psi\cos(\lambda-\lambda_S)}\frac{d}{dt}\phi \\ & +\left\{\frac{Q^2\cos^2\psi\cos(\lambda-\lambda_S)\sin(-\lambda+\lambda_S)\cos^3\beta}{\sin^2\psi(Q\sin\beta-\cos\psi\cos(\lambda-\lambda_S))}\right. \\ & \quad +\left[-\frac{Q\cos\psi\sin\beta\sin(-\lambda+\lambda_S)}{Q\sin\beta-\cos\psi\cos(\lambda-\lambda_S)}\right. \\ & \quad\quad \left.\left. +\left(-\frac{\cos\psi\cos^2(\lambda-\lambda_S)}{\sin^2\psi}+\frac{Q\sin\beta\cos(\lambda-\lambda_S)}{\sin^2\psi}\right)(\sin(-\lambda+\lambda_S))^{-1}\right]\cos\beta\right\}\frac{d}{dt}\lambda_S\end{aligned}$$

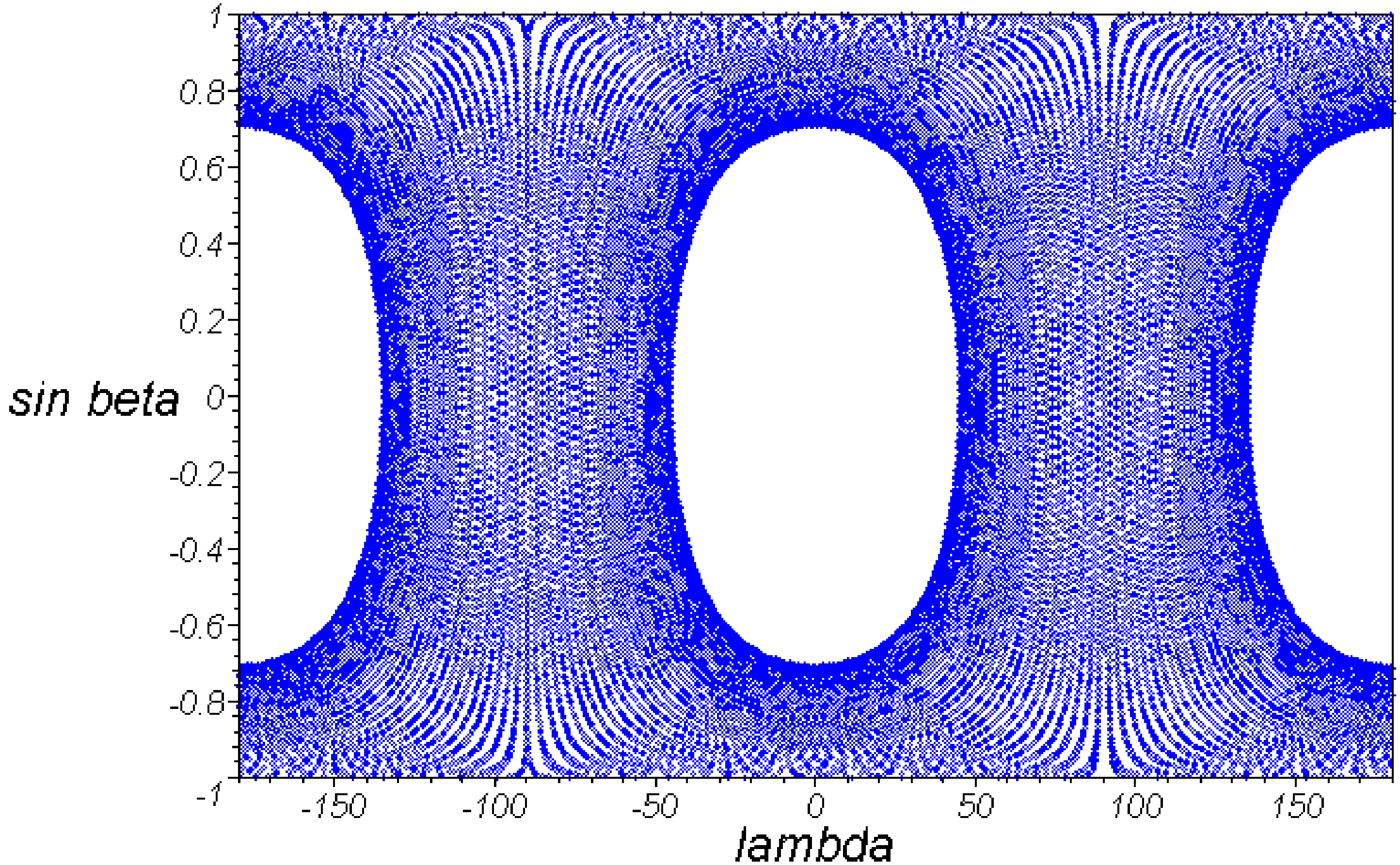
Rotation Rates *(continued)*

- Cross-scan rate Ω_C as a function of position on the sky
  - longitude coordinate is wrt Sun's ecliptic longitude
- Two solution surfaces due to Q
- Dominated by precession rate term
  - fortunately, an uncomplicated topology
- Ω_C determines the density of observations on the sky
  - **Expect a pile-up of density near the precession cone hole boundaries**
  - Will smear in longitude due to Earth's orbital motion
- plot below scaled by factor of 100 (arcsec/sec)



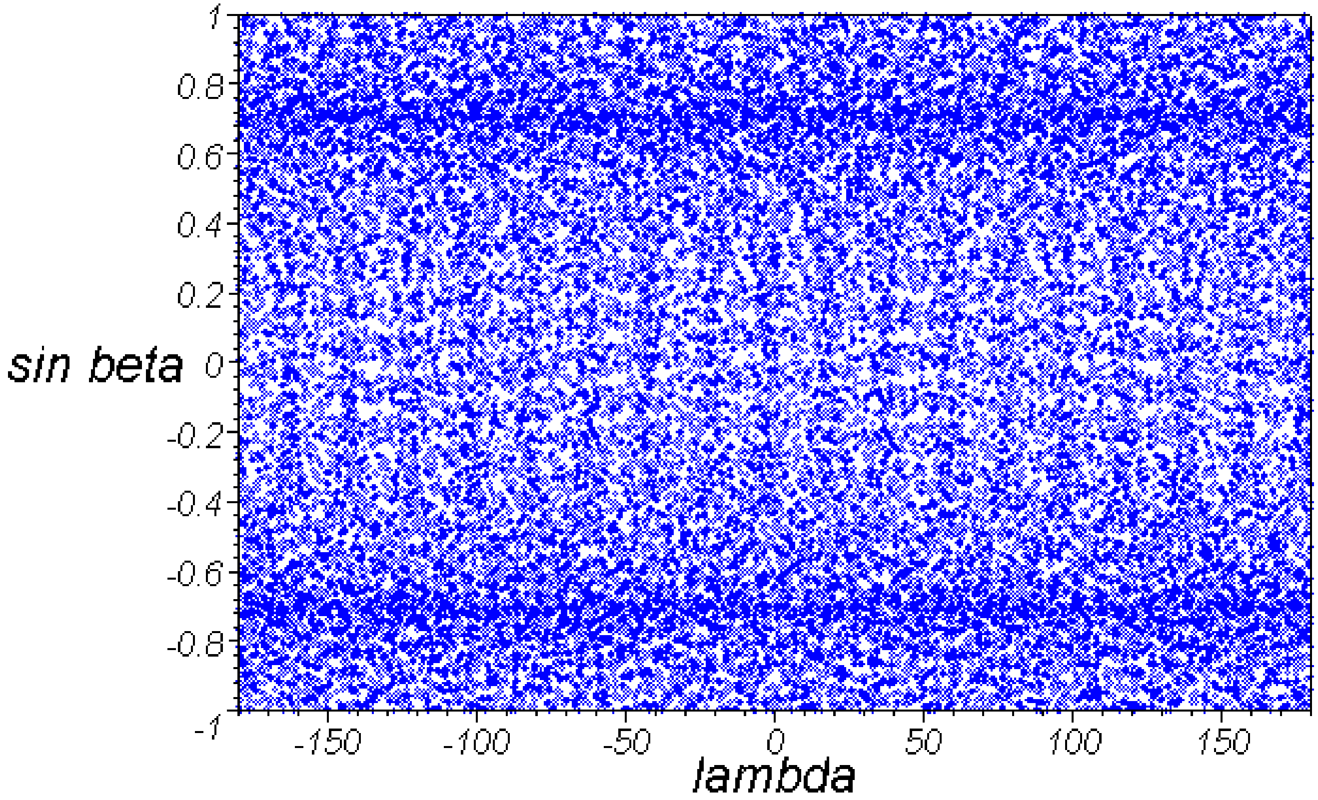
Simulations I. Distributions

- Observation density on the sky — comoving frame
  - observations every 21 minutes for 1 year
- Notice density enhancement near precession cone hole boundaries
- Use sin β to produce equal-area plot (Lambert cylindrical equal-area)



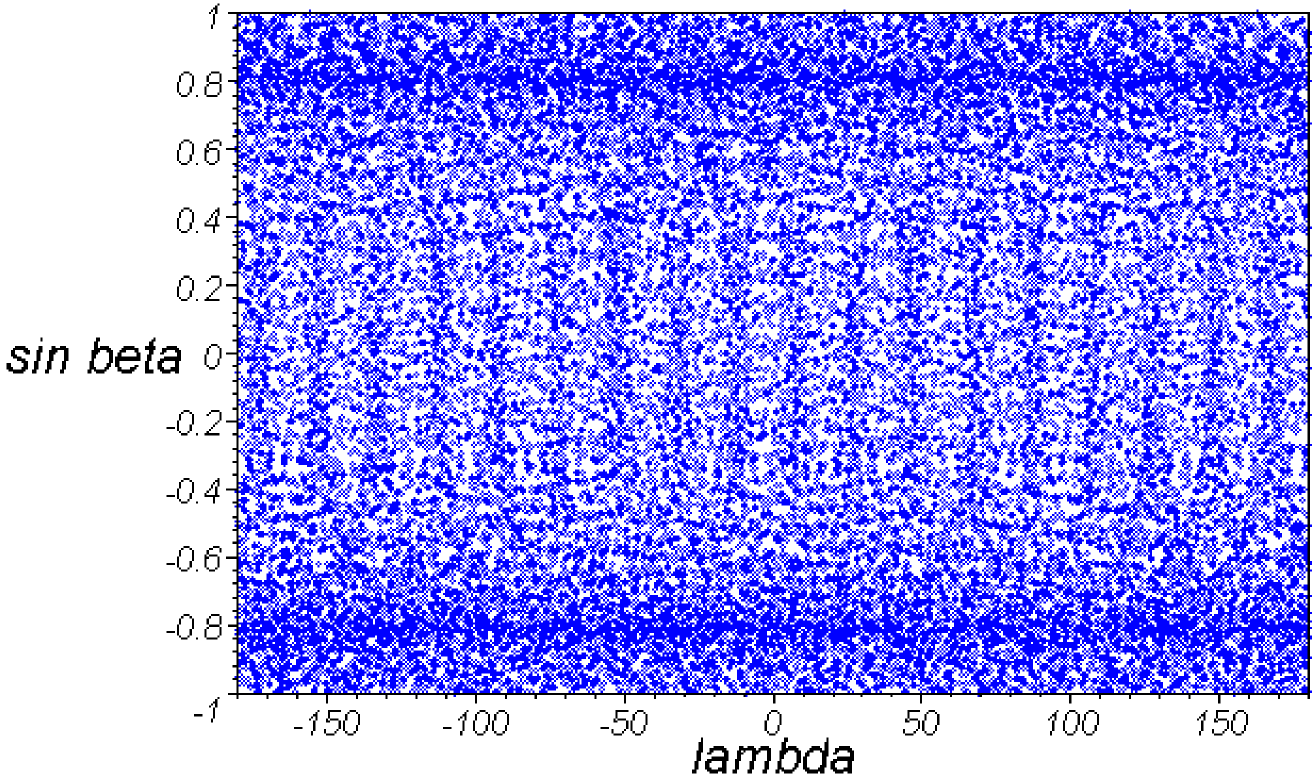
Simulations I. Distributions *(continued)*

- Observation density on the sky — true ecliptic frame
  - smearing in longitude
  - observations every 17.5 minutes for 1 year
- Sun angle = 45 degrees
- Notice density variation with latitude
- "Ribbing" in longitude is real (gets worse with slower precession)



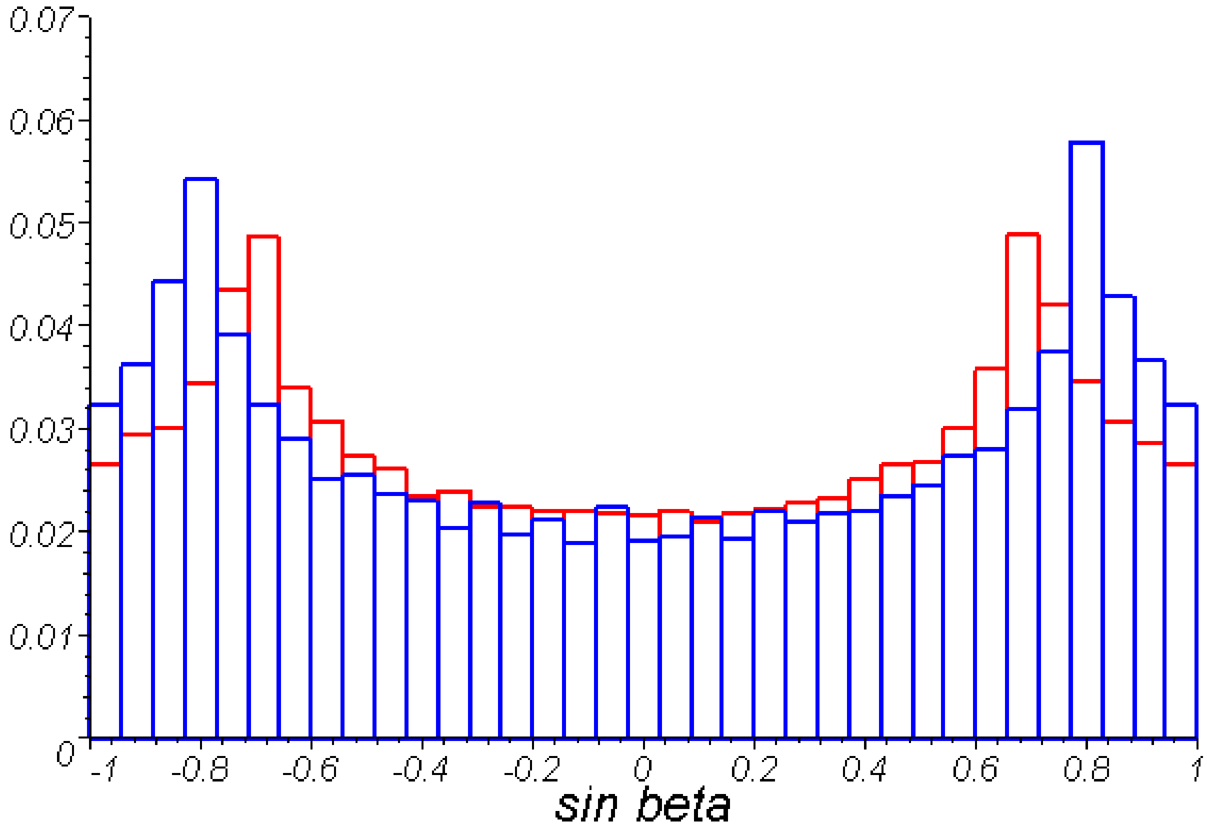
Simulations I. Distributions *(continued)*

- Observation density on the sky — true ecliptic frame
- Sun angle = 36 degrees
- Density enhancements have moved towards ecliptic poles, due to enlargement of precession cone holes
- Density peaks are higher, and trough regions are lower



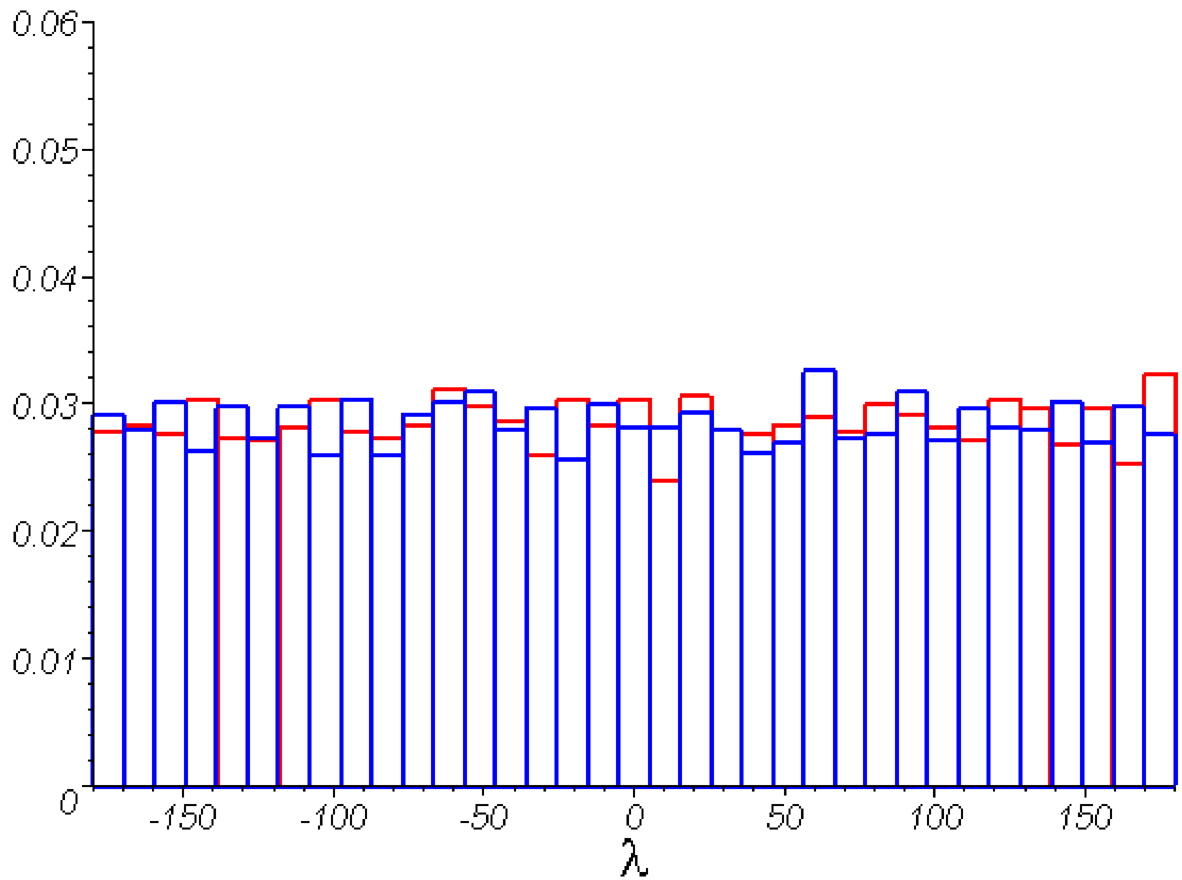
Simulations I. Distributions *(continued)*

- Observation density on the sky — true ecliptic frame
- Histograms (pdf) in sin β
  - blue = 36-degree Sun angle, red = 45-degree Sun angle
  - main feature: density pile-ups near precession cone hole boundaries
  - little difference between the two Sun angle cases



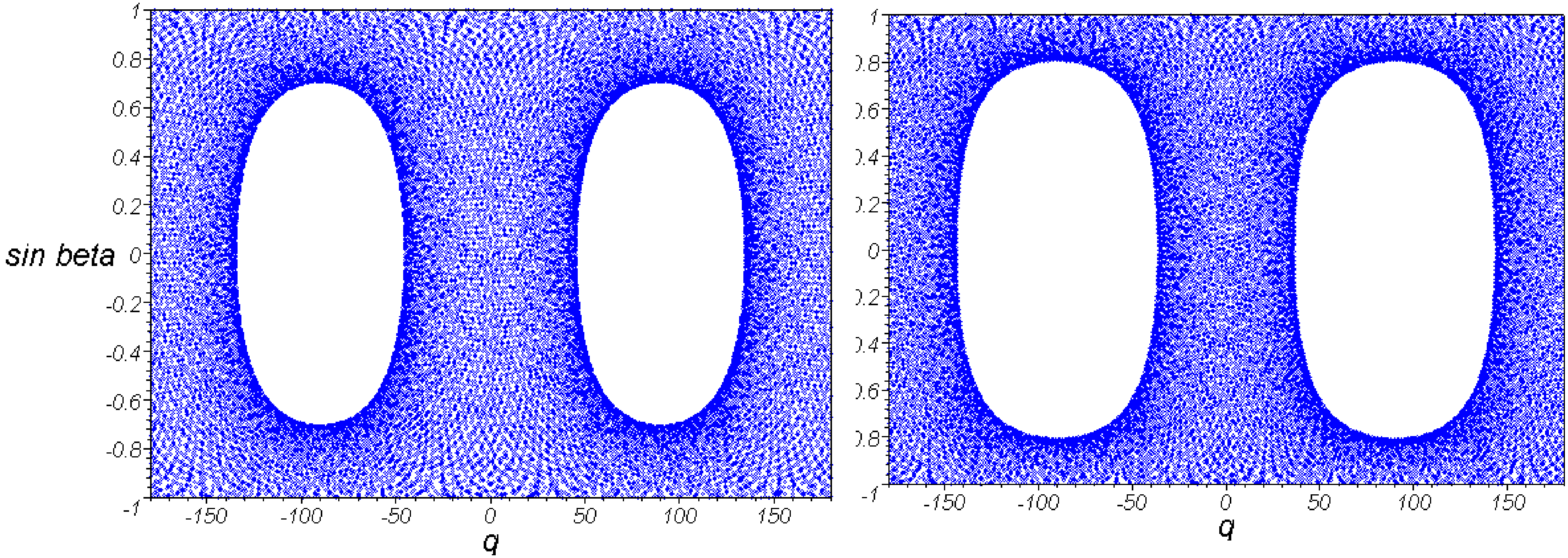
Simulations I. Distributions *(continued)*

- Observation density on the sky — true ecliptic frame
- Histograms (pdf) in λ
  - blue = 36-degree Sun angle, red = 45-degree Sun angle
  - main feature: pretty flat across longitude — smearing is pretty good after 1 year
  - however, "ribbing" in longitude is real and gets worse with increasing precession period



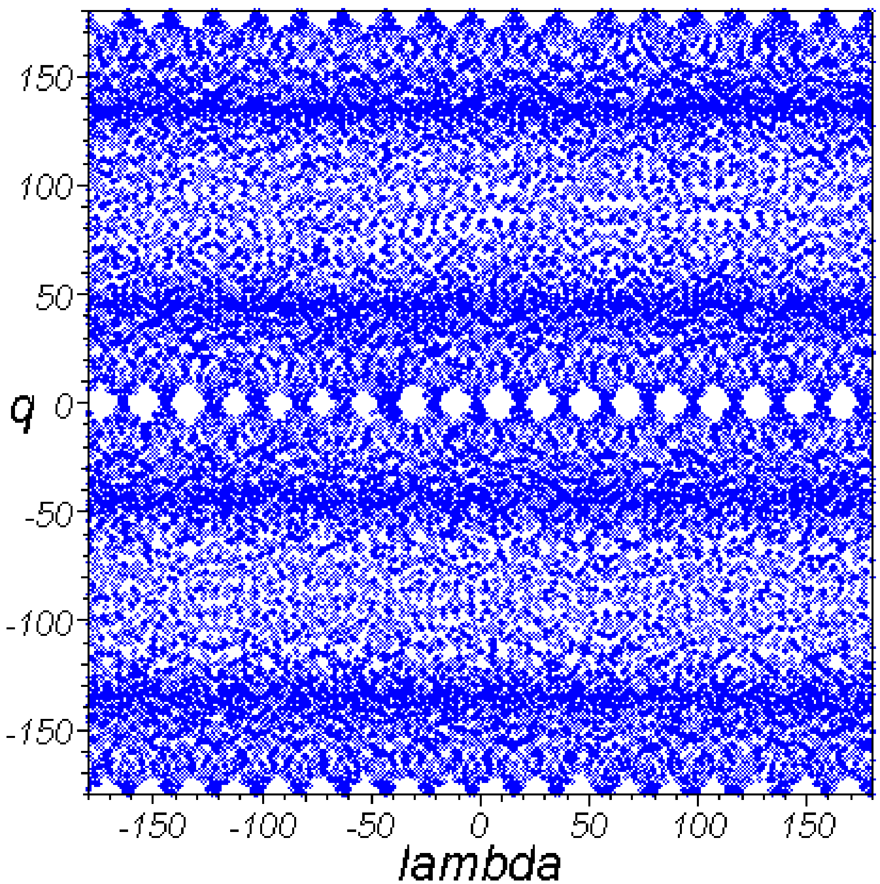
Simulations I. Distributions *(continued)*

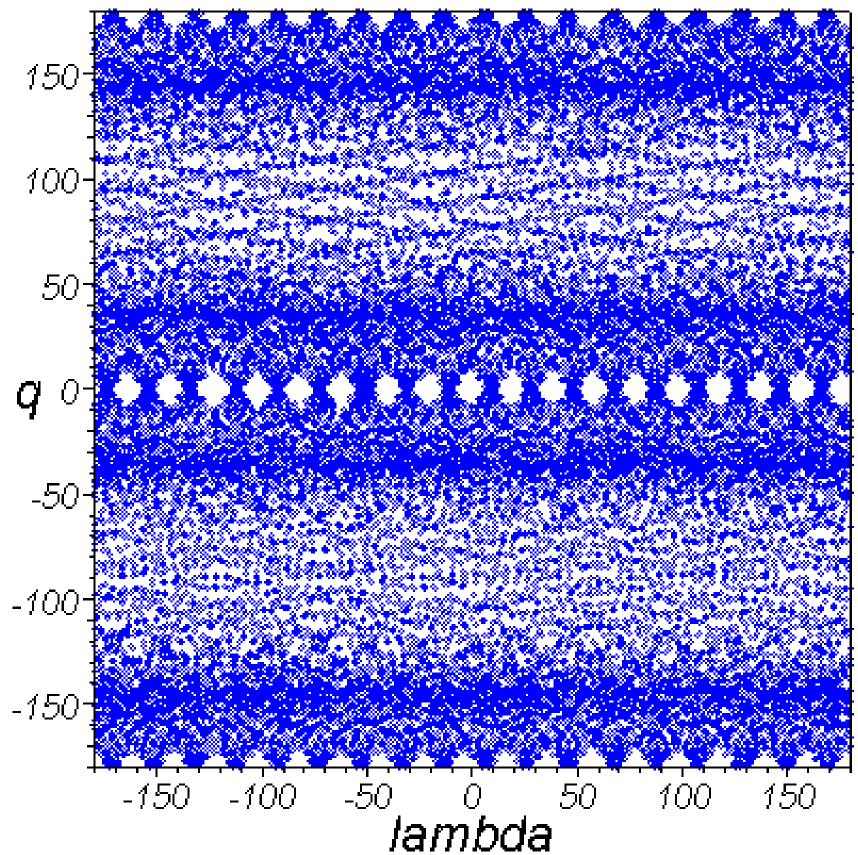
- Scan angle distribution — comoving frame
  - as function of sin β
  - again, density enhancements near precession cone hole boundaries
  - left plot: ψ = 45 deg   right plot: ψ = 36 deg
  - hole size increases as Sun angle decreases



Simulations I. Distributions *(continued)*

- Scan angle distribution — comoving frame
  - as function of λ
  - again, density enhancements near precession cone hole boundaries
  - left plot: ψ = 45 deg   right plot: ψ = 36 deg
  - density enhancements migrate as Sun angle changes



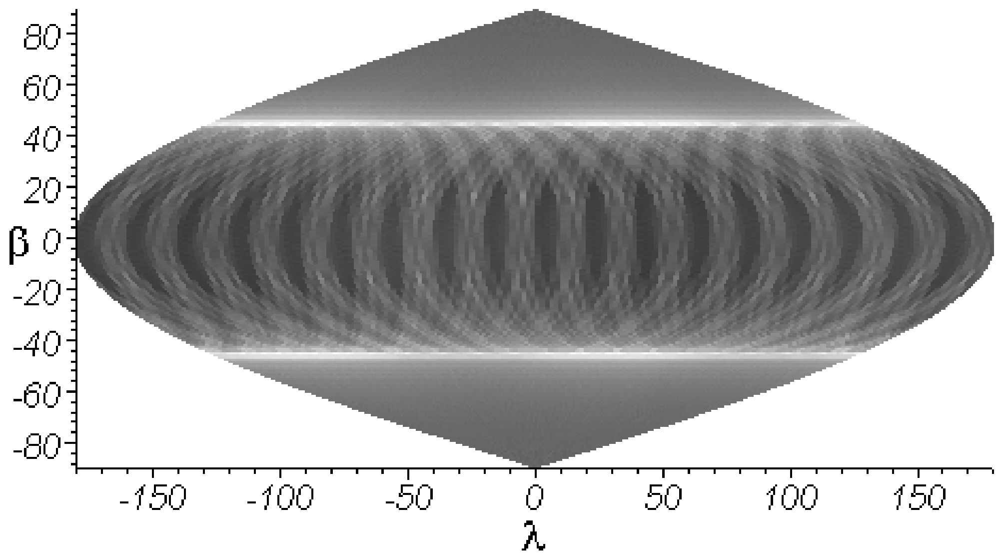


Simulations II. Observations on an Equal-Area Grid

- This set of simulations consisted of 6 runs
  - ψ = 36, 45, 54 deg
  - T_φ = 20, 30 days
  - Only (ψ,T_φ)=(36,20) and (ψ,T_φ)=(45,20) cases shownhere for brevity.
- Simulation length: 2.5 years
- Observation frequency: every 4.2 seconds
- Observation errors sampled from Gaussian error distributions
  - along-scan 1σ error: 0.6 mas
  - cross-scan 1σ error: 10 mas
- Observations accumulated on equal-area [λ,sin β] grid
  - grid dimensions = [120,96]
- Statistical quantities calculated for each grid cell
  - observation error scale (essentially 1/sqrt(N))
  - scan angle q
  - ecliptic latitude & longitude
  - parallax

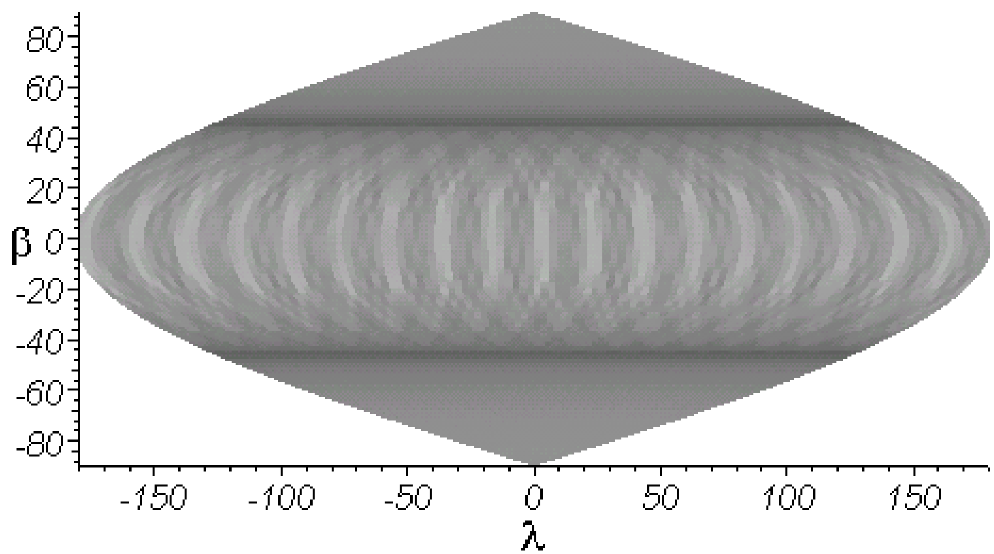
Raw Counts (45 deg)

- Note:
  - region of crummy errors grows in area with decrease of Sun angle
  - longitudinal "ribbing"
  - (Sinusoidal equal-area projection)



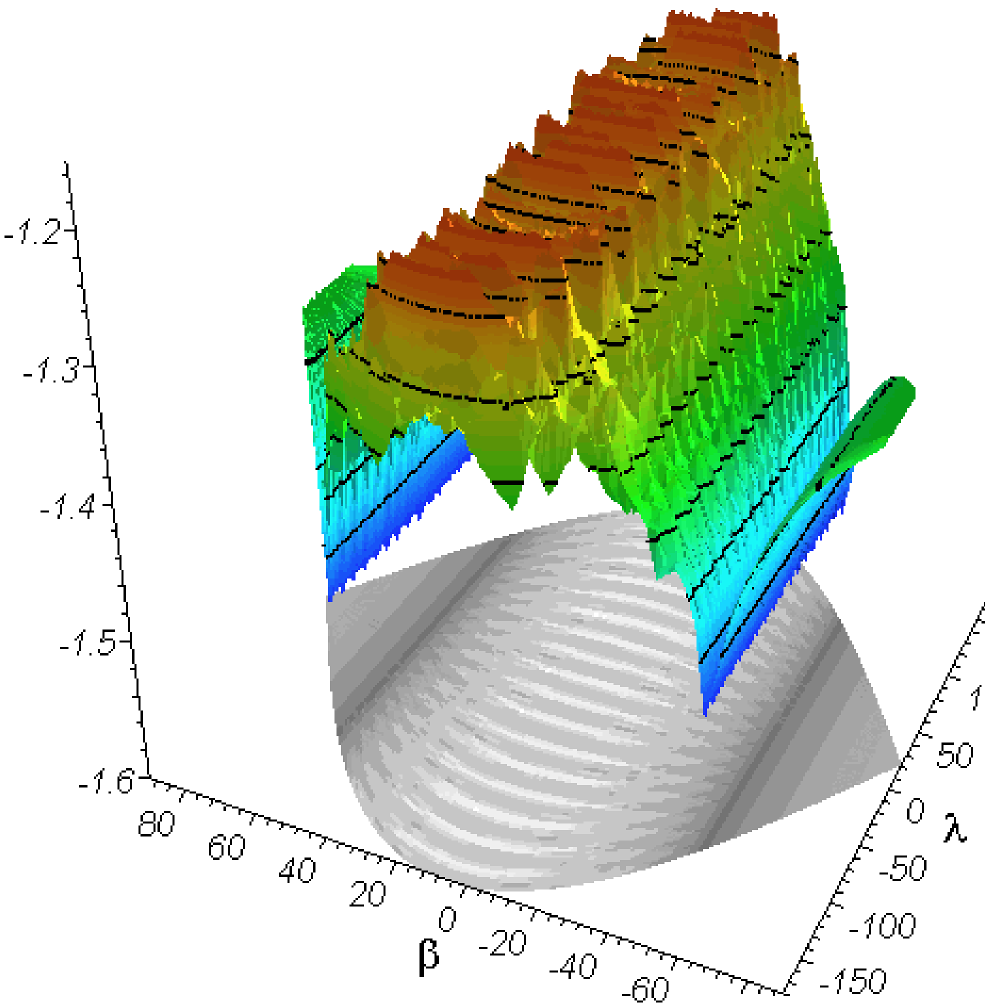
Error Scaling

- Observation error scaling
  - proportional to log 1/sqrt(N) in each cell
  - magnitude scaled to 1000 observations per cell
  - (typical ranges of N were actually ~1000 to ~3000 per cell)
- Note:
  - accuracy enhancements at density pile-ups

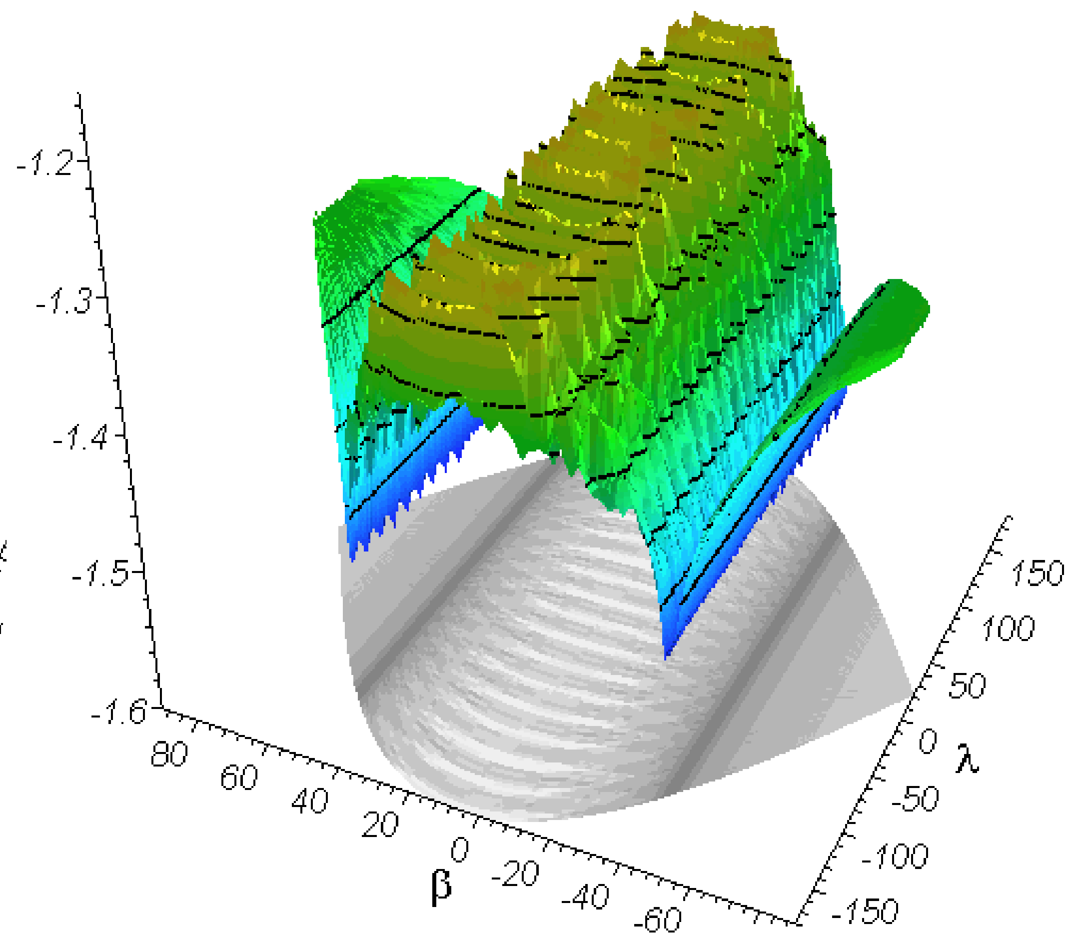


Error Scaling *(continued)*

- ψ = 36 deg

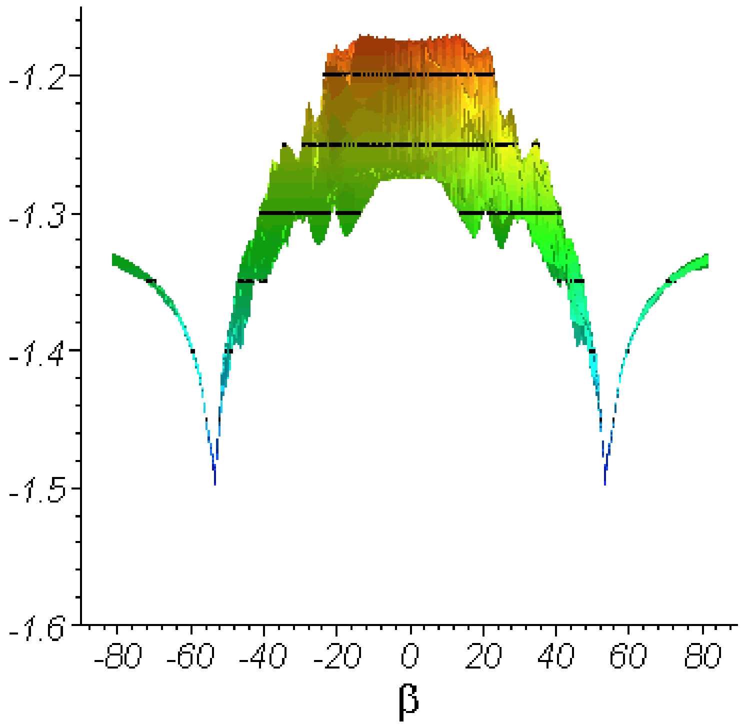


- ψ = 45 deg

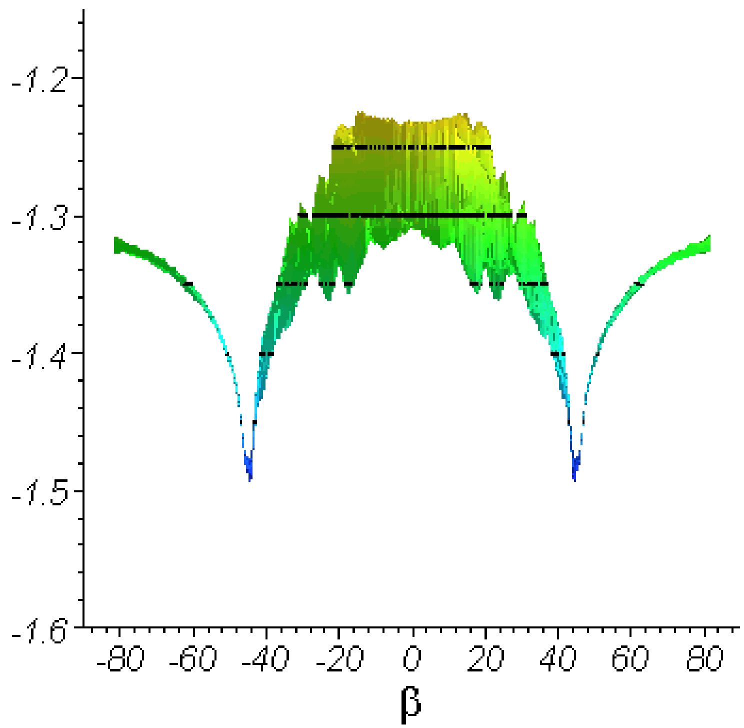


Error Scaling *(continued)*

- ψ = 36 deg

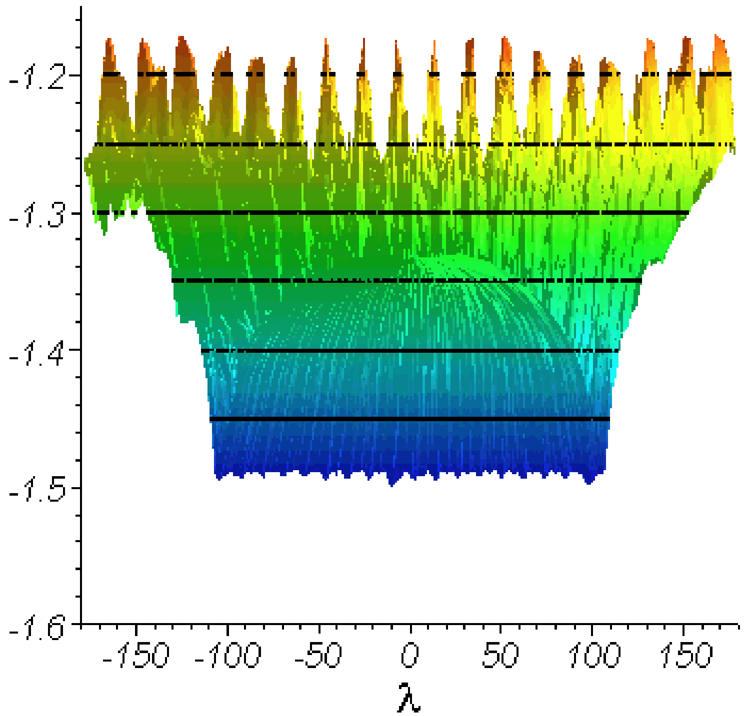


- ψ = 45 deg

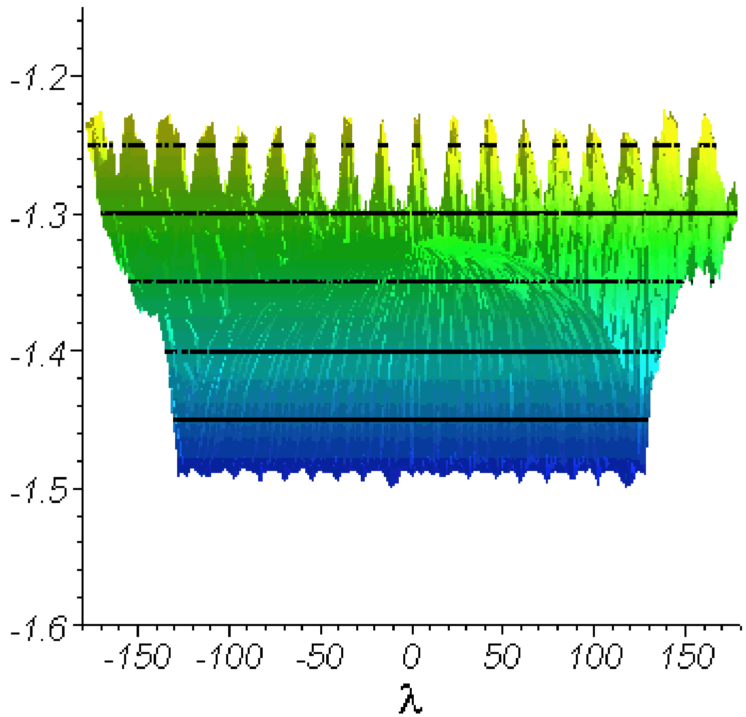


Error Scaling *(continued)*

- ψ = 36 deg

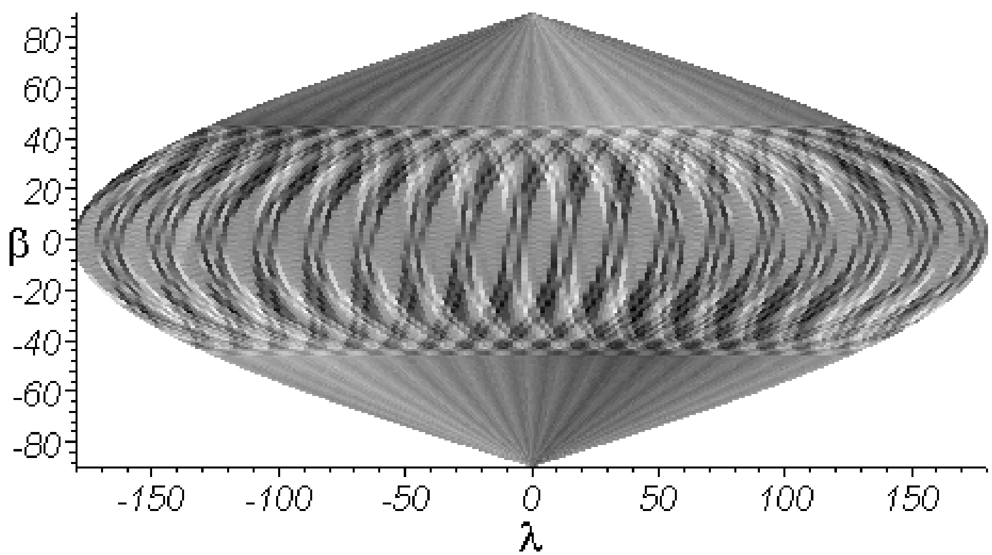


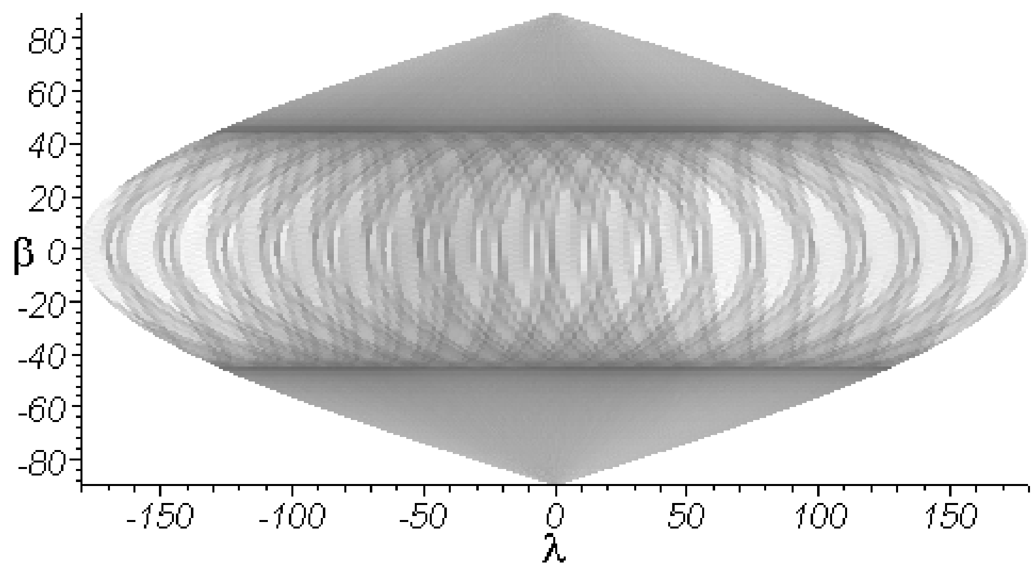
- ψ = 45 deg



Scan Angle

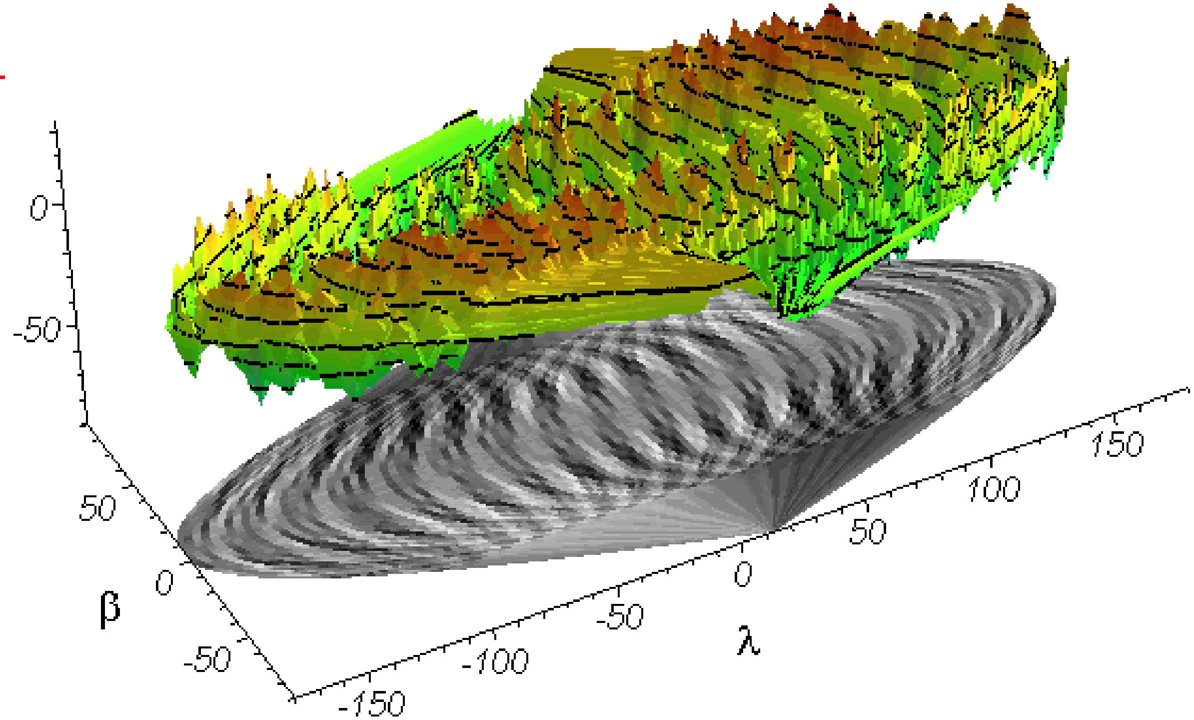
- mean q in each grid cell
  - range: -41 to +41 deg
- 1σ errors in mean q
  - range: 2.2 to 5.5 deg



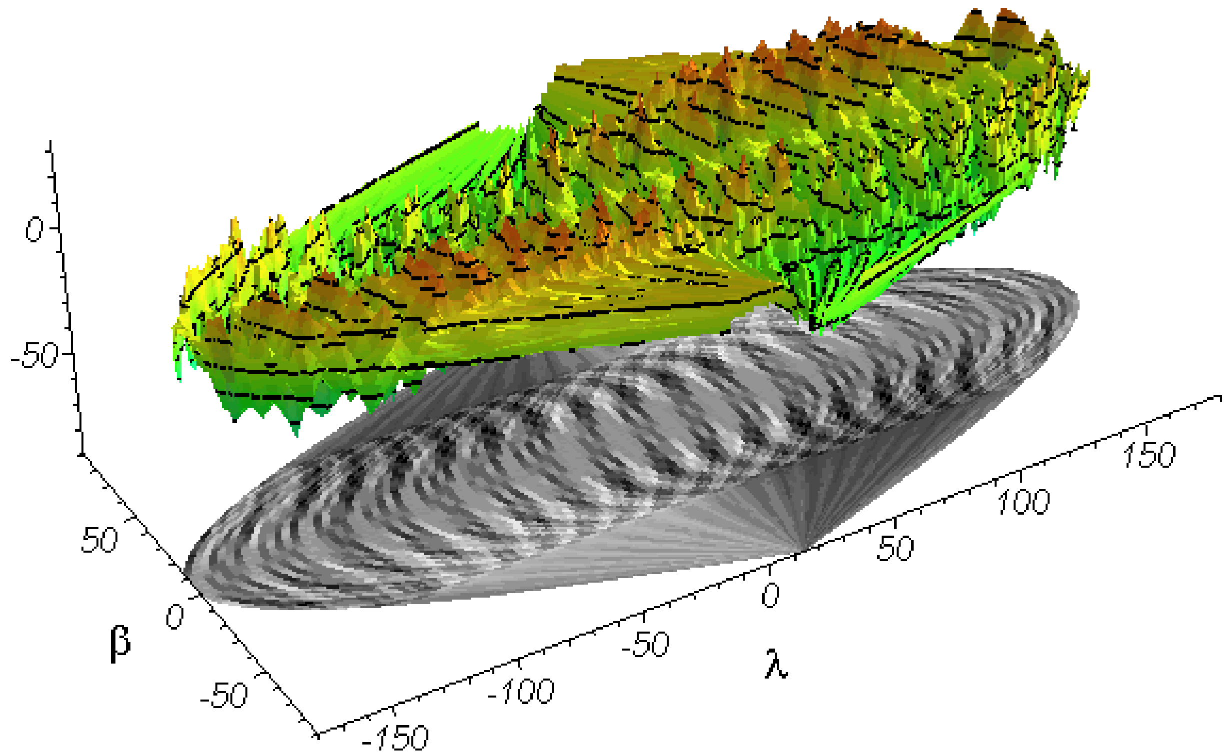


Scan Angle

- ψ = 36 deg

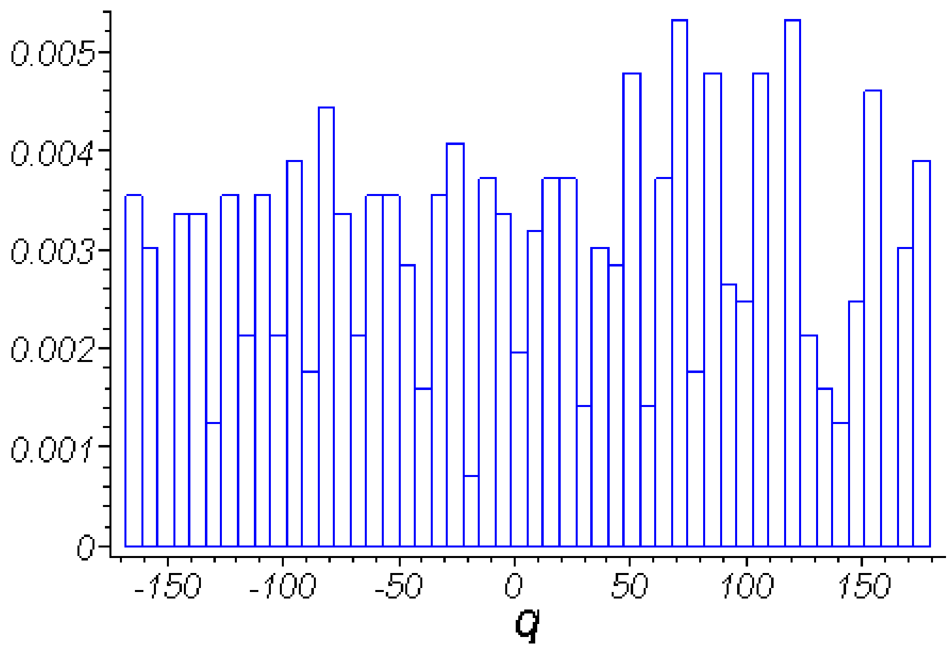


- ψ = 45 deg

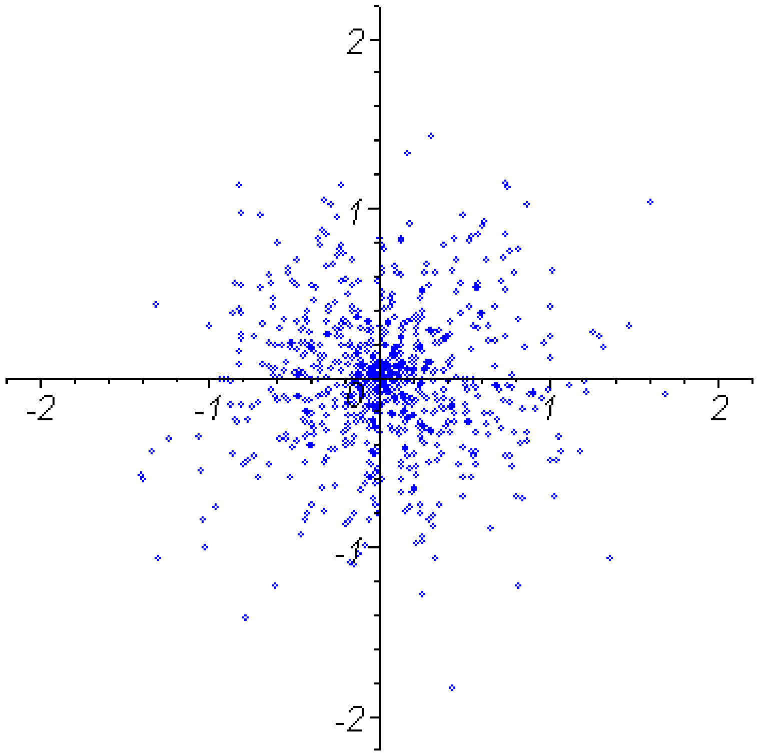


Scan Angle Distribution in One Cell (Example 1)

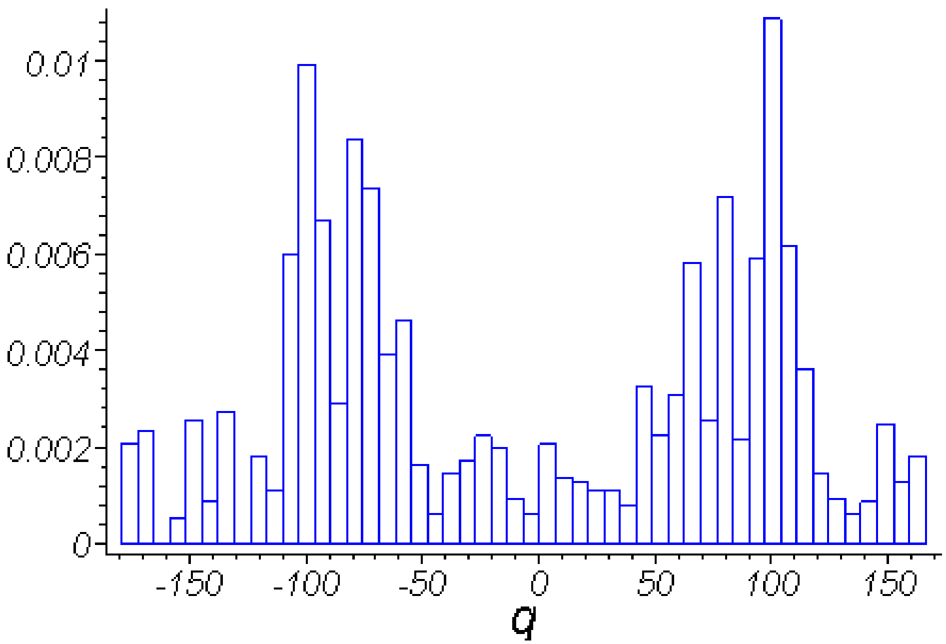
- probability distribution function

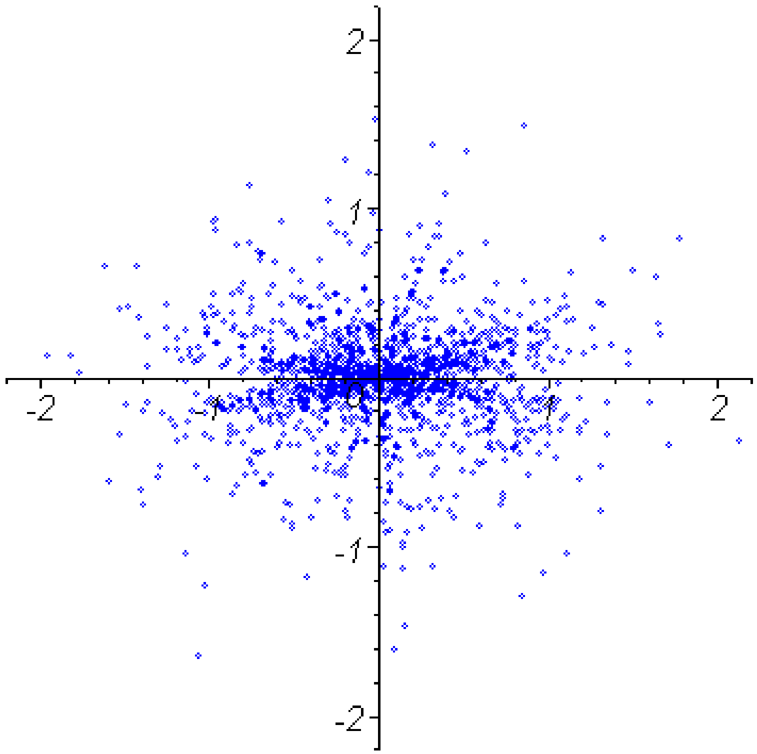


- scatterplot in ecliptic coordinates
  - scale is in milliarcseconds
  - along-scan single-observation errors: Gaussian distribution with $1\sigma = 0.6$ mas

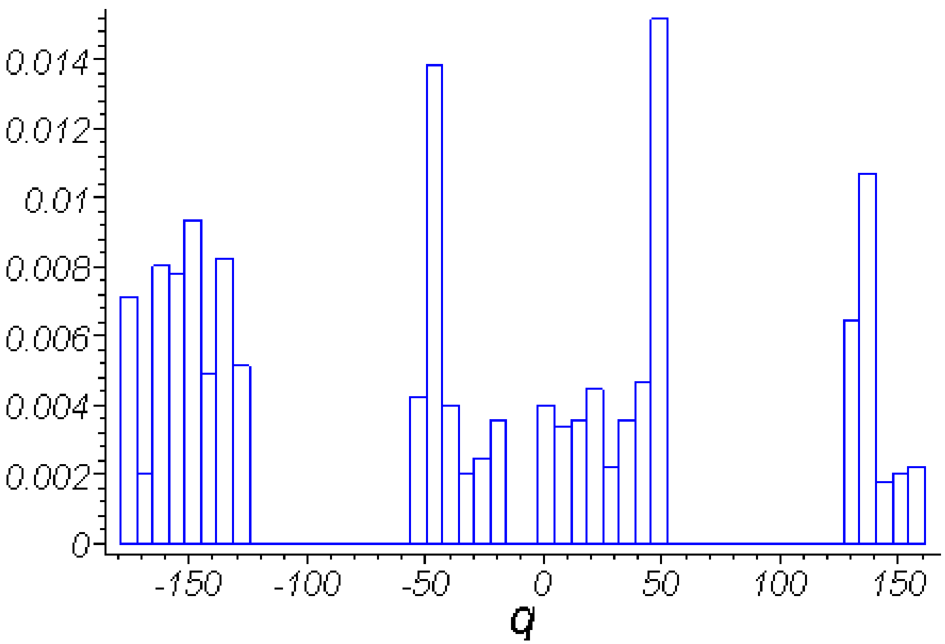


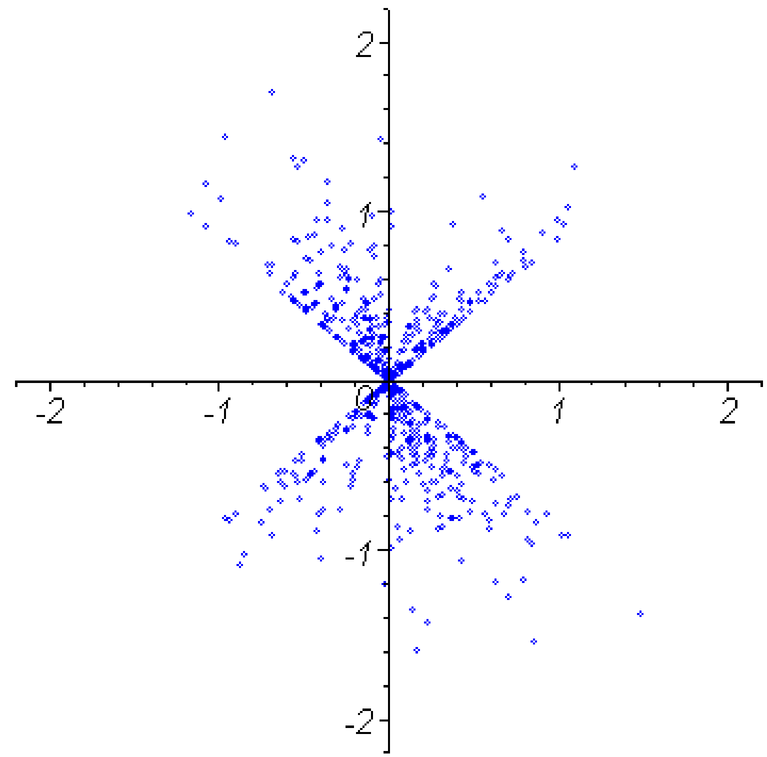
Scan Angle Distribution in One Cell (Example 2)





Scan Angle Distribution in One Cell (Example 3)





Summary of Results

- Distributions are determined by certain 3D surfaces as functions of ecliptic coordinates
- Sampling on these surfaces as s/c scans the sky creates the two distributions
  - Surface geometry is complicated and very nonuniform (this is bad)
  - Surfaces are slowly smeared in ecliptic longitude as Earth orbits the Sun (this is good)
  - Unfortunately, there is no smearing of the surfaces in ecliptic latitude, so we're stuck with the effects of surface variation as a function of latitude
- Observation density distribution
  - Two zones in latitude where the density peaks
    - Corresponding depression of mean errors
  - Dependence on Sun angle
    - Zones move to higher latitude with smaller Sun angle
    - Zone overdensity increases with smaller Sun angle
  - Dependence on precession period
    - longitudinal inhomogeneities get worse with larger precession period

Summary of Results *(continued)*

- Scan angle distribution
  - Large-scale latitude zones (due to precession cone hole effect)
  - Distribution shape:
    - *highly* dependent on sky location
    - ranges from nearly-Gaussian radial profile to *highly* non-Gaussian
  - Dependence on Sun angle:
    - Usual high-latitude zone motion
    - Longitudinal inhomogeneities still large after 2.5 years
  - Dependence on precession period:
    - inhomogeneities get worse with larger precession period
- Ratios of sky-averaged error scaling
  - For comparison with HIPPARCOS simulations
    - latitude — 20 deg : 30 deg : 40 deg = 1.13 : 1.00 : 0.97
    - longitude — 20 deg : 30 deg : 40 deg = 1.70 : 1.00 : 0.76
  - However, all-sky averages mask inhomogeneities ("ribbing")
  - Dependence on Sun angle:
    - 36 deg : 45 deg : 54 deg = 1.08 : 1.00 : 0.98
  - Dependence on precession period:
    - 45 deg: 30 days / 20 days = 0.99

What's Next?

- Calculate astrometric parameter errors
- Calculate useful metrics
  - Contours of percentage of stars
    - To do right, requires convolving actual stellar distribution
    - Rough cut: assume homogeneous stellar distribution
    - Better: stellar distribution models
      - Work well except for Galactic plane region
  - Isolate ecliptic plane region (region of worst errors and worst inhomogeneities)
  - Characterize scan angle distributions
- Text, text, and more text (technical memoranda)