Graphical Portraits of Selected Coordinate Systems

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Contents

This document contains simple 2D and 3D plots representing certain selected coordinate systems. In each 3D plot are three surfaces, each a constant over one of the coordinates. The 2D plots are two sets of coordinate curves for a given value of the third coordinate – that is, an intersection of a specific coordinate surface with families of the other two coordinate surfaces. The coordinate systems are a small subset of ones known by Maple; they were chosen for my own peculiar reasons and are therefore not representative of anything in particular. The Maple coordinate systems are listed in the help for coords.

Click on a link to go directly to that section. Alternately, use the bookmarks for navigation (under the View menu item).

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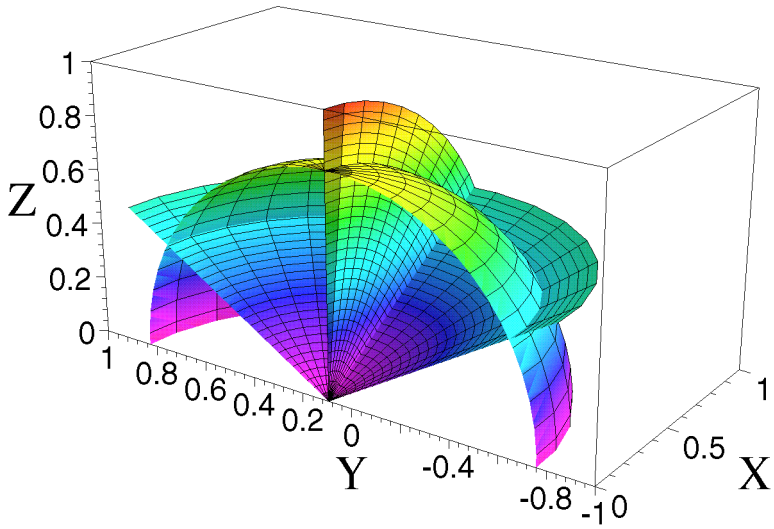
Spherical

```
spherical:
  x = u*cos(v)*sin(w)
  y = u*sin(v)*sin(w)
  z = u*cos(w)
```

$xyz := \mathrm{fn}([u\cos(v)\sin(w), u\sin(v)\sin(w), u\cos(w)], u, v, w)$

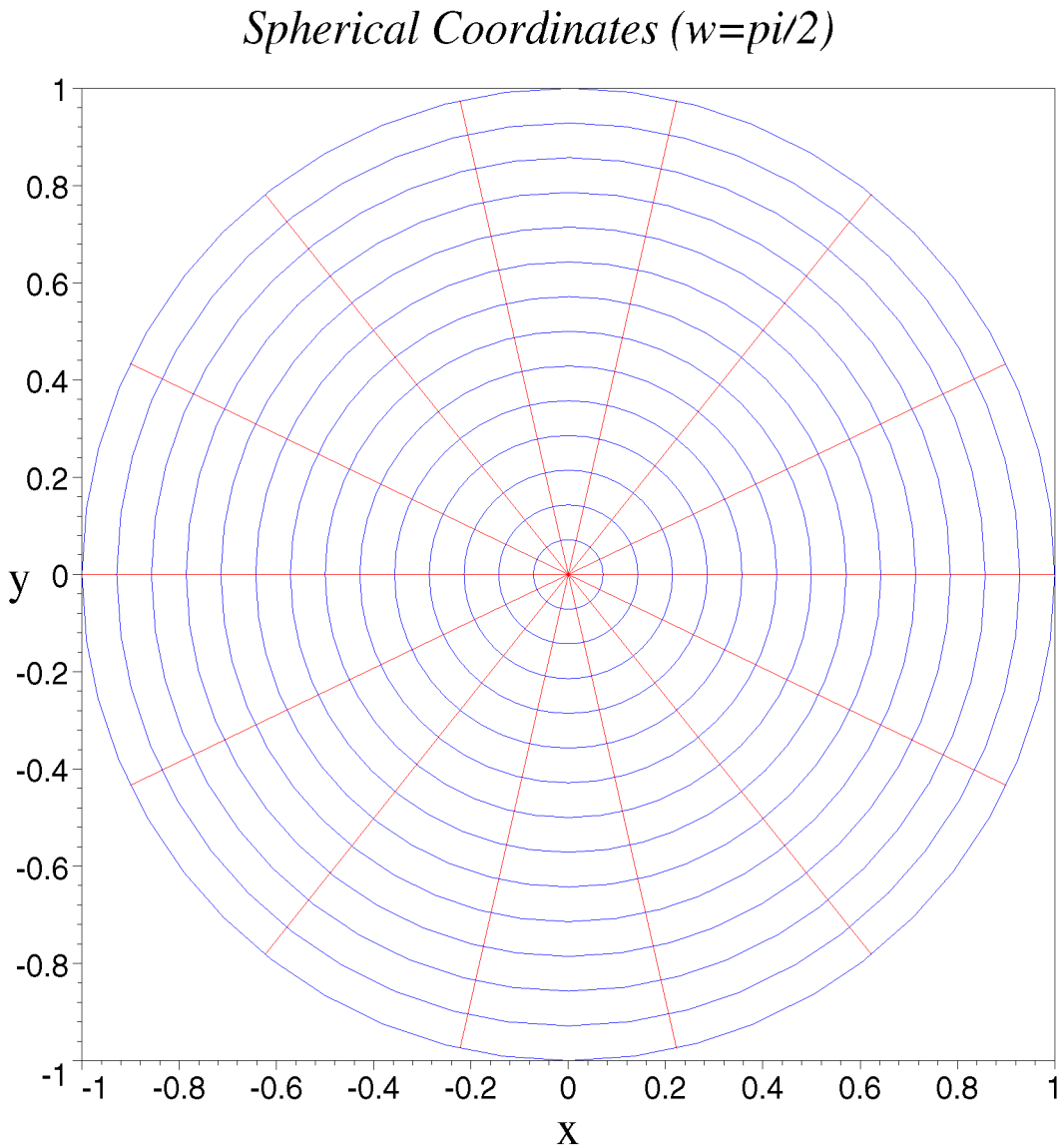
$$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = .8, v_0 = 0, w_0 = \mathrm{evalf}\left(\frac{\pi}{3}\right), u = 0 .. 1, v = -\pi .. \pi, w = 0 .. \pi, orientation = [-145, 70], view = [0 .. 1, -1 .. 1, 0 .. 1], title = \text{"Spherical Coordinates"}\right)$$

Spherical Coordinates



$xy := \mathrm{fn}([u\cos(v), u\sin(v)], u, v)$

coordplot2D(xy(u, v), $u = 0 .. 1$, $v = -\pi .. \pi$, 15, *view* = [−1 .. 1, −1 .. 1], *title* = "Spherical Coordinates (w=pi/2)")



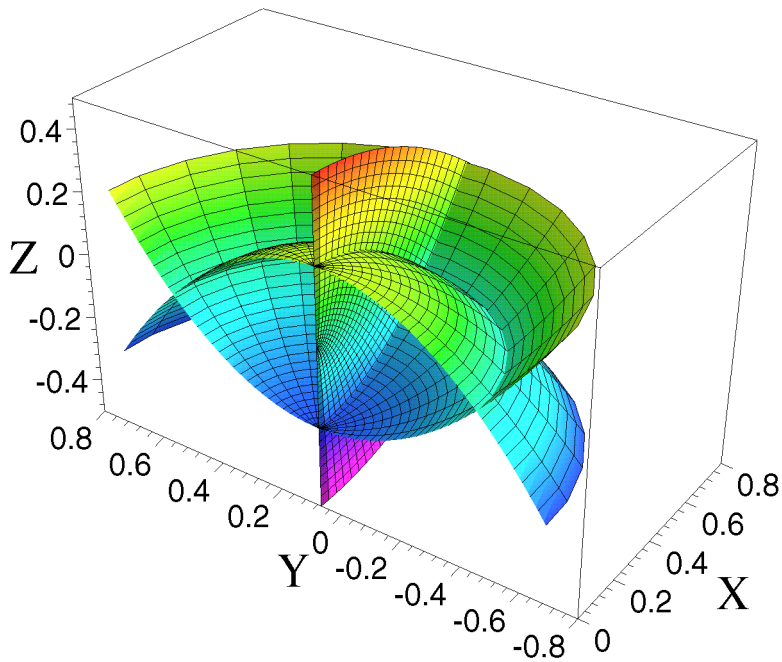
Paraboloidal

```
paraboloidal:                    ( Spiegel )
  x = u*v*cos(w)
  y = u*v*sin(w)
  z = (u^2 - v^2)/2
```

$$xyz := \mathrm{fn}\left(\left[u\,v\cos(w),\ u\,v\sin(w),\ \frac{u^2 - v^2}{2}\right], u, v, w\right)$$

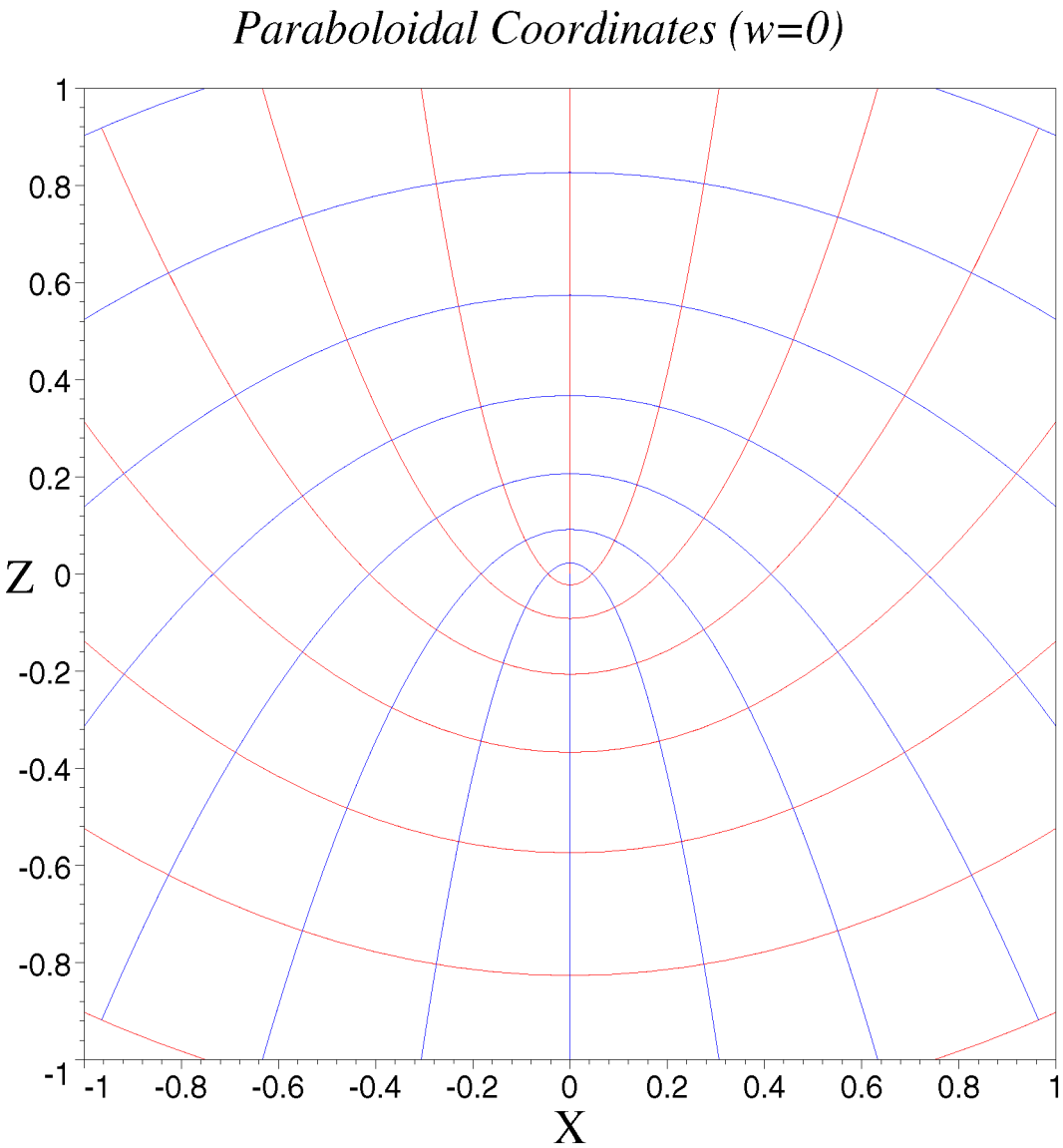
coordplot3D(xyz(u, v, w), $u_0 = .7$, $v_0 = .7$, $w_0 = 0$, $u = 0 .. 1$, $v = 0 .. 1$, $w = -\pi .. \pi$,
orientation = [−145, 60], *view* = [0 .. .8, −.8 .. .8, −.5 .. .5], *title* = "Paraboloidal Coordinates")

Paraboloidal Coordinates



$$xz := \mathrm{fn}\left(\left[u\,v, \frac{1\,u^2}{2} - \frac{1\,v^2}{2}\right], u, v\right)$$

coordplot2D(xz(u, v), $u = -1.5$ .. 1.5, $v = -1.5$ .. 1.5, 15, *view* = [−1 .. 1, −1 .. 1], *title* = "Paraboloidal Coordinates (w=0)", *labels* = ["X", "Z"])



Oblate Spheroidal

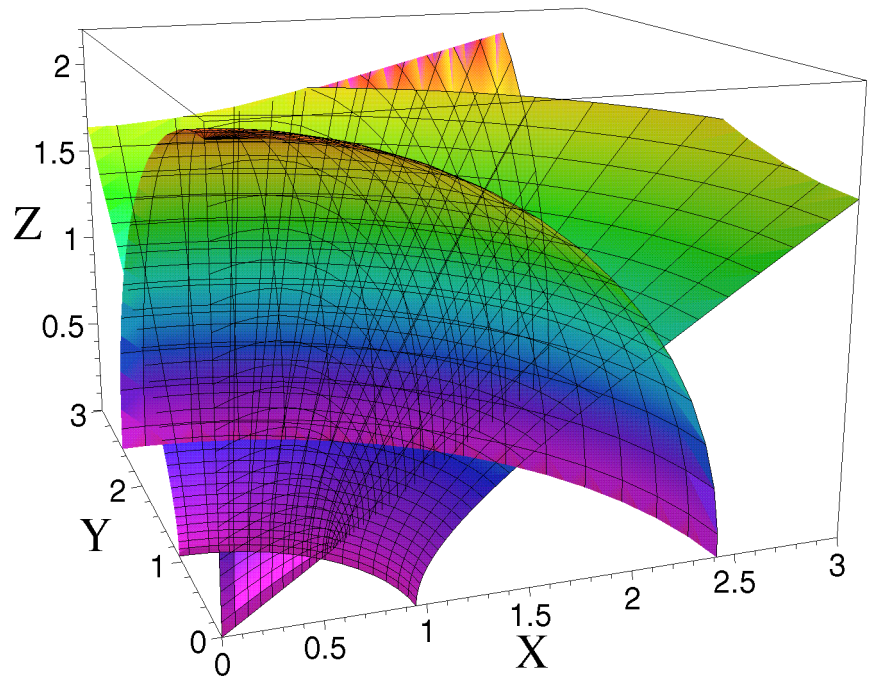
```
oblatespheroidal:
 x = a*cosh(u)*sin(v)*cos(w)
 y = a*cosh(u)*sin(v)*sin(w)
 z = a*sinh(u)*cos(v)
```

$xyz := \mathrm{fn}([\cosh(u)\sin(v)\cos(w), \cosh(u)\sin(v)\sin(w), \sinh(u)\cos(v)], u, v, w)$

$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = 1.5, v_0 = \mathrm{evalf}\left(\frac{\pi}{3}\right), w_0 = \mathrm{evalf}\left(\frac{\pi}{4}\right), u = 0 .. 2, v = 0 .. \pi, w = 0 .. \pi,\right.$

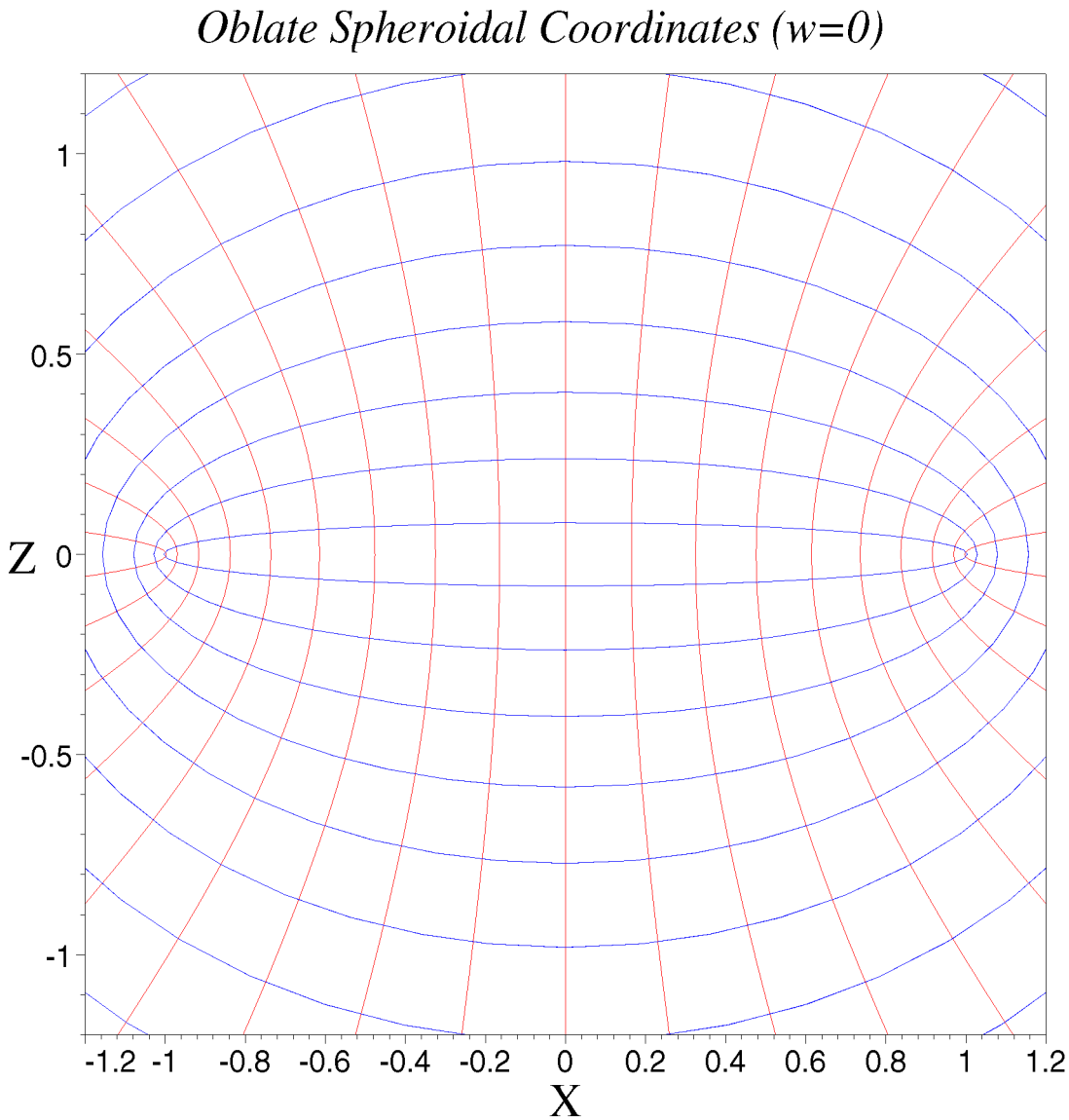
$\left. orientation = [-110, 75], view = [0 .. 3, 0 .. 3, 0 .. 2.2], title = \text{"Oblate Spheroidal Coordinates"}\right)$

Oblate Spheroidal Coordinates



$xz := \mathrm{fn}([\cosh(u)\sin(v), \sinh(u)\cos(v)], u, v)$

coordplot2D(xz(u, v), $u = -1.5 .. 1.5$, $v = -\pi .. \pi$, 20, *view* = [−1.2 .. 1.2, −1.2 .. 1.2], *title* = "Oblate Spheroidal Coordinates (w=0)", *labels* = ["X", "Z"])



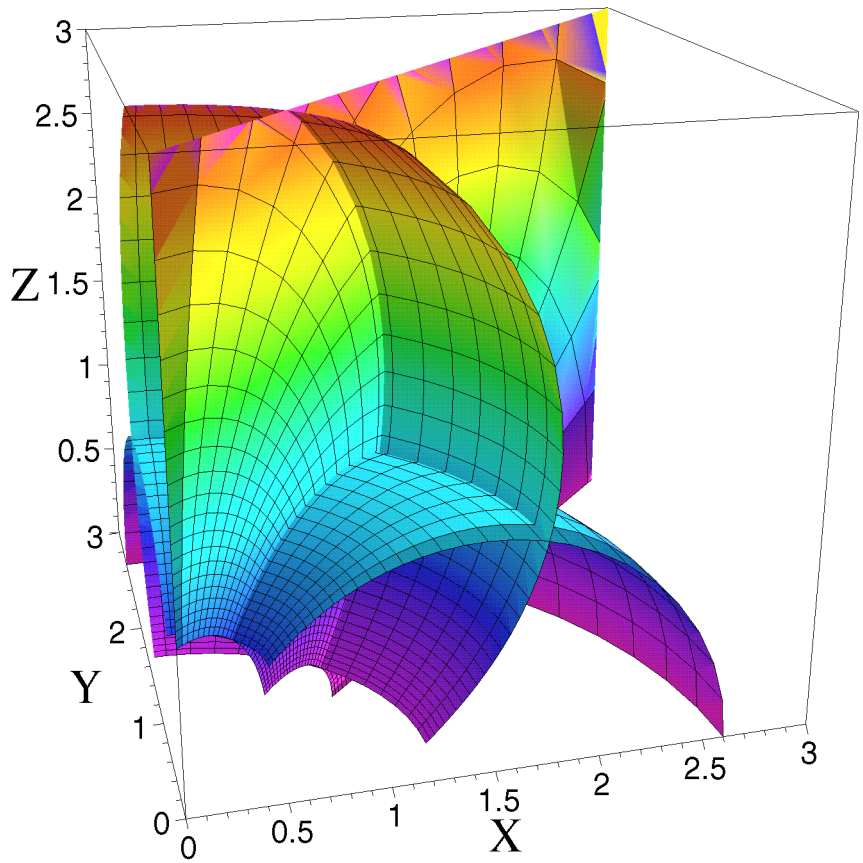
Inverse Oblate Spheroidal

```
invoblspheroidal:            (inverse oblate spheroidal)
  x = a*cosh(u)*sin(v)*cos(w)/(cosh(u)^2-cos(v)^2)
  y = a*cosh(u)*sin(v)*sin(w)/(cosh(u)^2-cos(v)^2)
  z = a*sinh(u)*cos(v)/(cosh(u)^2-cos(v)^2)
```

$$xyz := \mathrm{fn}\left(\left[\frac{\cosh(u)\sin(v)\cos(w)}{\cosh(u)^2-\cos(v)^2}, \frac{\cosh(u)\sin(v)\sin(w)}{\cosh(u)^2-\cos(v)^2}, \frac{\sinh(u)\cos(v)}{\cosh(u)^2-\cos(v)^2}\right], u, v, w\right)$$

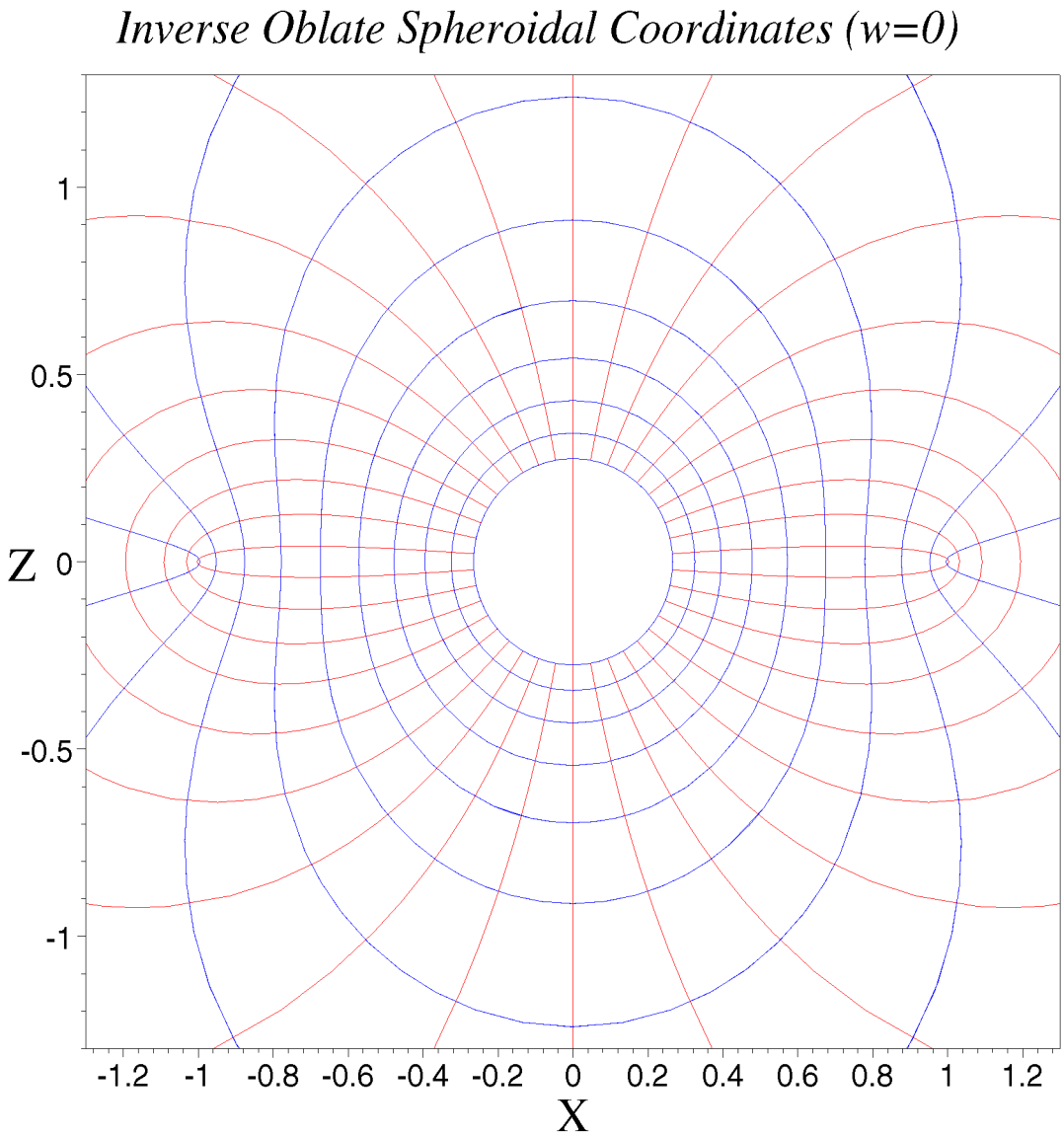
$$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = .3, v_0 = .4, w_0 = \mathrm{evalf}\left(\frac{\pi}{4}\right), u = 0 .. 1, v = 0 .. 1, w = 0 .. \pi,\right.$$
$$\left. orientation = [-105, 70], view = [0 .. 3, 0 .. 3, 0 .. 3], title = \text{"Inverse Oblate Spheroidal Coordinates"}\right)$$

Inverse Oblate Spheroidal Coordinates



$$xz := \mathrm{fn}\left(\left[\frac{\cosh(u)\sin(v)}{\cosh(u)^2 - \cos(v)^2}, \frac{\sinh(u)\cos(v)}{\cosh(u)^2 - \cos(v)^2}\right], u, v\right)$$

coordplot2D(xz(u, v), $u = -2 .. 2$, $v = -\pi .. \pi$, 20, *view* = [−1.3 .. 1.3, −1.3 .. 1.3], *title* = "Inverse Oblate Spheroidal Coordinates (w=0)", *labels* = ["X", "Z"])



Prolate Spheroidal

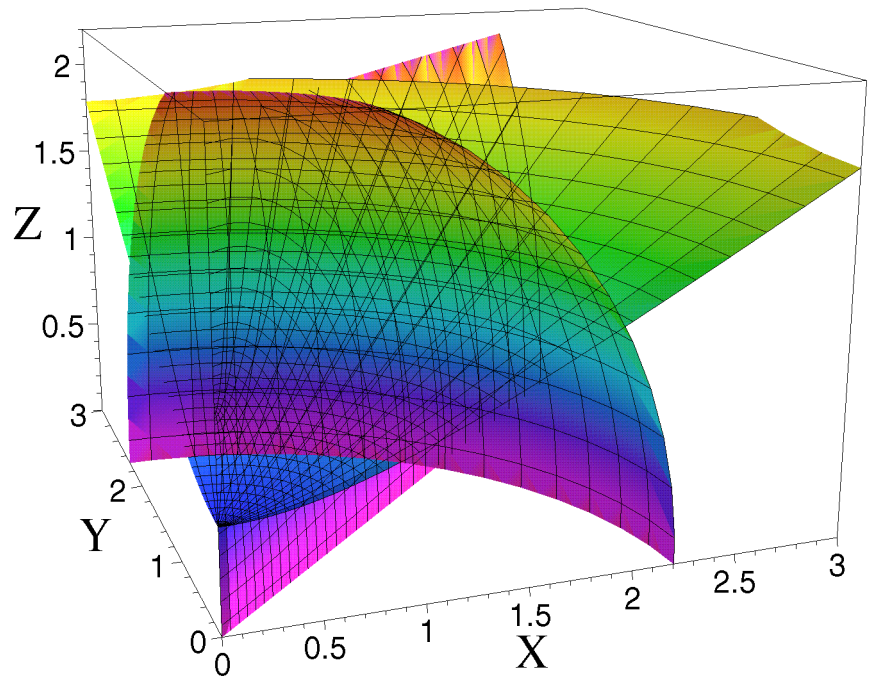
```
prolatespheroidal:
  x = a*sinh(u)*sin(v)*cos(w)
  y = a*sinh(u)*sin(v)*sin(w)
  z = a*cosh(u)*cos(v)
```

$xyz := \mathrm{fn}([\sinh(u)\sin(v)\cos(w), \sinh(u)\sin(v)\sin(w), \cosh(u)\cos(v)], u, v, w)$

$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = 1.5, v_0 = \mathrm{evalf}\left(\frac{\pi}{3}\right), w_0 = \mathrm{evalf}\left(\frac{\pi}{4}\right), u = 0 .. 2, v = 0 .. \pi, w = 0 .. \pi,\right.$

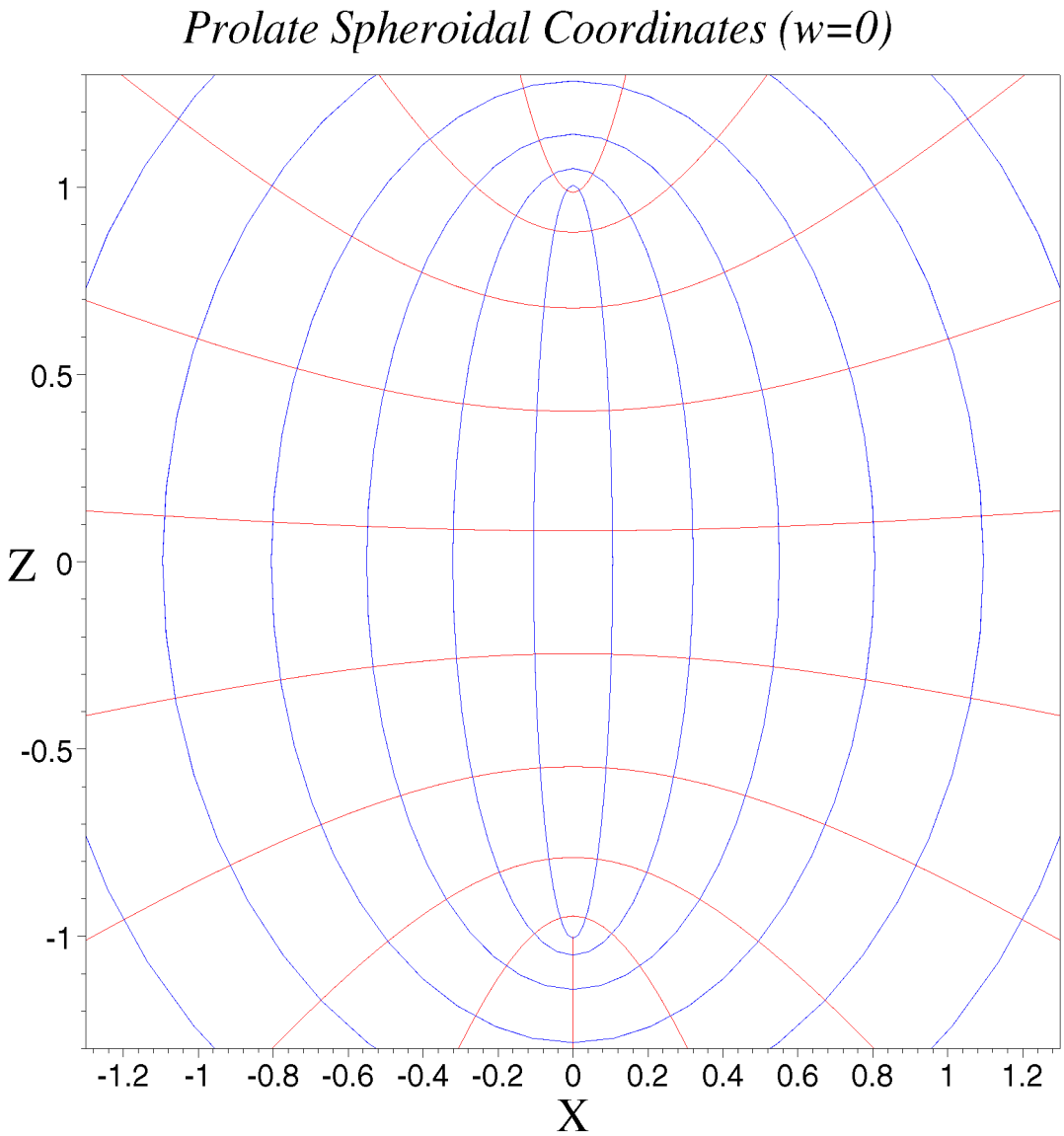
$\left. orientation = [-110, 75], view = [0 .. 3, 0 .. 3, 0 .. 2.2], title = \text{"Prolate Spheroidal Coordinates"}\right)$

Prolate Spheroidal Coordinates



$xz := \mathrm{fn}([\sinh(u)\sin(v), \cosh(u)\cos(v)], u, v)$

$\mathrm{coordplot2D}(\mathrm{xz}(u, v), u = -2 .. 2, v = -\pi .. \pi, 20, view = [-1.3 .. 1.3, -1.3 .. 1.3],$
$title =$ "Prolate Spheroidal Coordinates (w=0)", $labels = [$"X", "Z"$])$



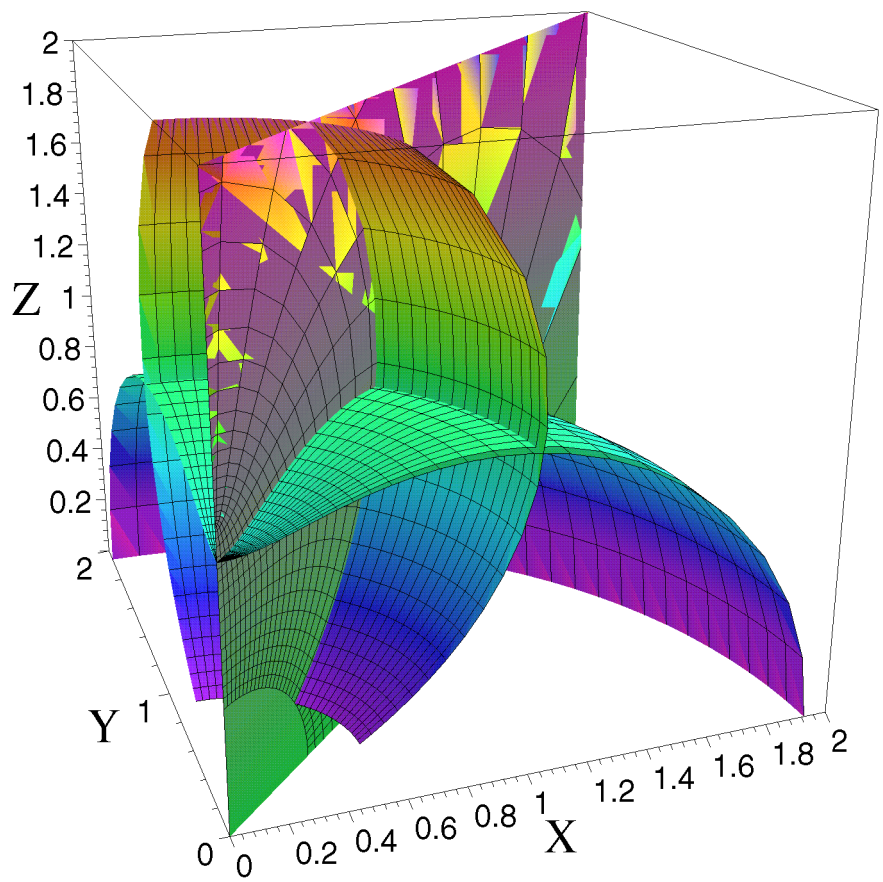
Inverse Prolate Spheroidal

```
invprospheroidal:            (inverse prolate spheroidal)
  x = a*sinh(u)*sin(v)*cos(w)/(cosh(u)^2-sin(v)^2)
  y = a*sinh(u)*sin(v)*sin(w)/(cosh(u)^2-sin(v)^2)
  z = a*cosh(u)*cos(v)/(cosh(u)^2-sin(v)^2)
```

$$xyz := \mathrm{fn}\left(\left[\frac{\sinh(u)\sin(v)\cos(w)}{\cosh(u)^2-\sin(v)^2}, \frac{\sinh(u)\sin(v)\sin(w)}{\cosh(u)^2-\sin(v)^2}, \frac{\cosh(u)\cos(v)}{\cosh(u)^2-\sin(v)^2}\right], u, v, w\right)$$

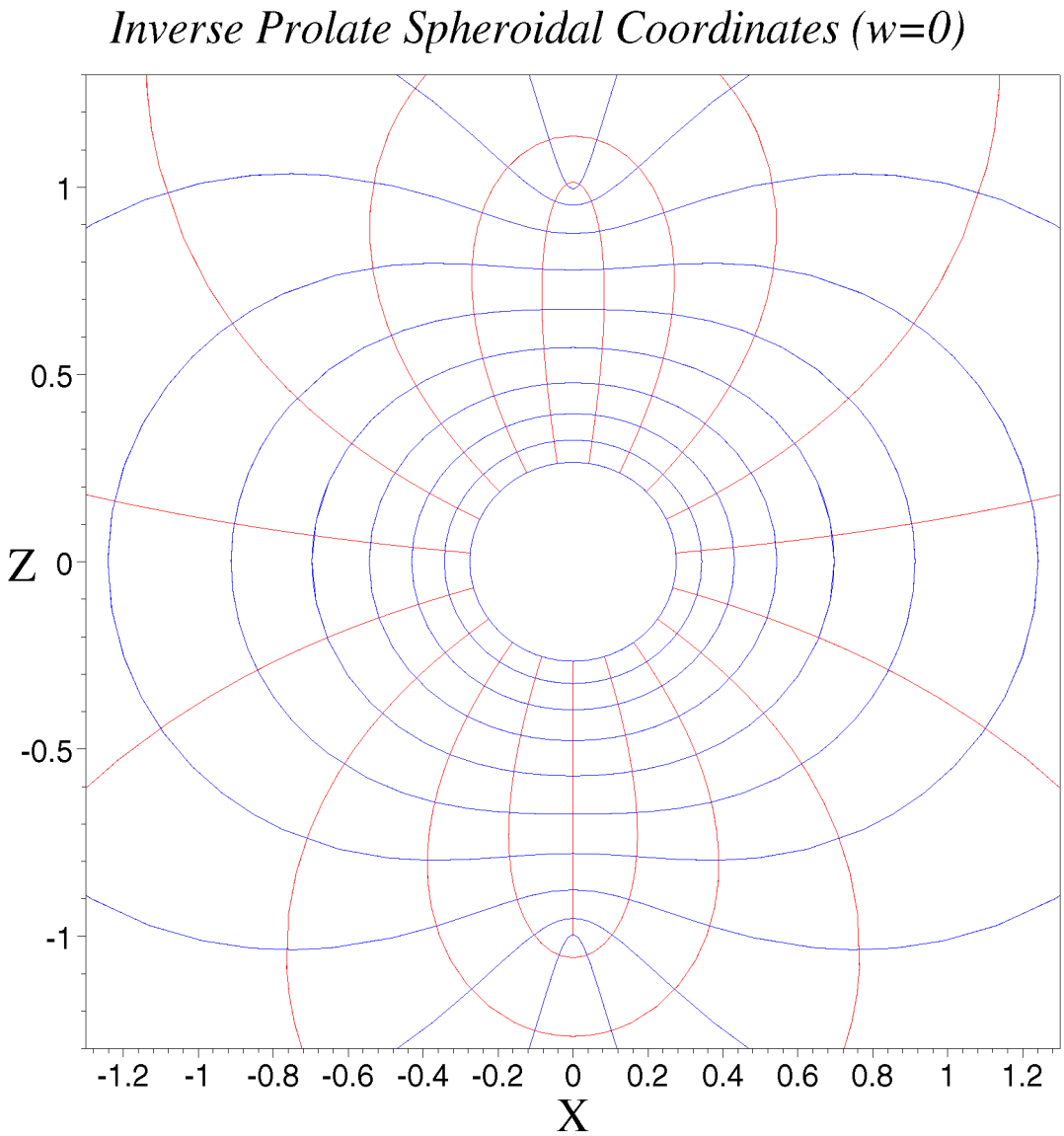
$$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = .5, v_0 = 1.1, w_0 = \mathrm{evalf}\left(\frac{\pi}{4}\right), u = -1 .. 1.5, v = 0 .. \frac{\pi}{2}, w = 0 .. \frac{\pi}{2},\right.$$
$$\left. orientation = [-110, 70], view = [0 .. 2, 0 .. 2, 0 .. 2], title = \text{"Inverse Prolate Spheroidal Coordinates"}\right)$$

Inverse Prolate Spheroidal Coordinates



$$xz := \mathrm{fn}\left(\left[\frac{\sinh(u)\sin(v)}{\cosh(u)^2 - \sin(v)^2}, \frac{\cosh(u)\cos(v)}{\cosh(u)^2 - \sin(v)^2}\right], u, v\right)$$

coordplot2D(xz(u, v), $u = -2$ .. 2, $v = -\pi$ .. π, 20, *view* = [−1.3 .. 1.3, −1.3 .. 1.3],
title = "Inverse Prolate Spheroidal Coordinates (w=0)", *labels* = ["X", "Z"])



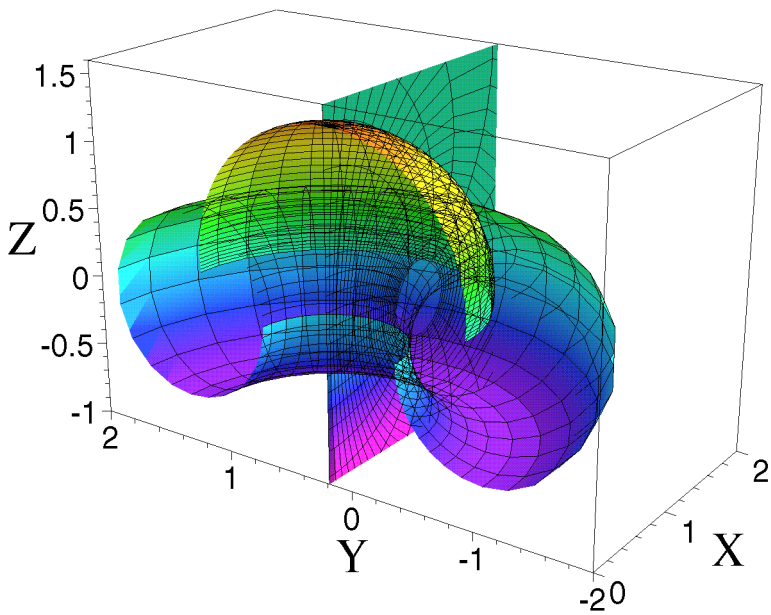
Toroidal

```
toroidal:
  x = a*sinh(v)*cos(w)/d
  y = a*sinh(v)*sin(w)/d
  z = a*sin(u)/d              ( where d = cosh(v) - cos(u) )
```

$$xyz := \mathrm{fn}\left(\left[\frac{\sinh(v)\cos(w)}{\cosh(v)-\cos(u)}, \frac{\sinh(v)\sin(w)}{\cosh(v)-\cos(u)}, \frac{\sin(u)}{\cosh(v)-\cos(u)}\right], u, v, w\right)$$

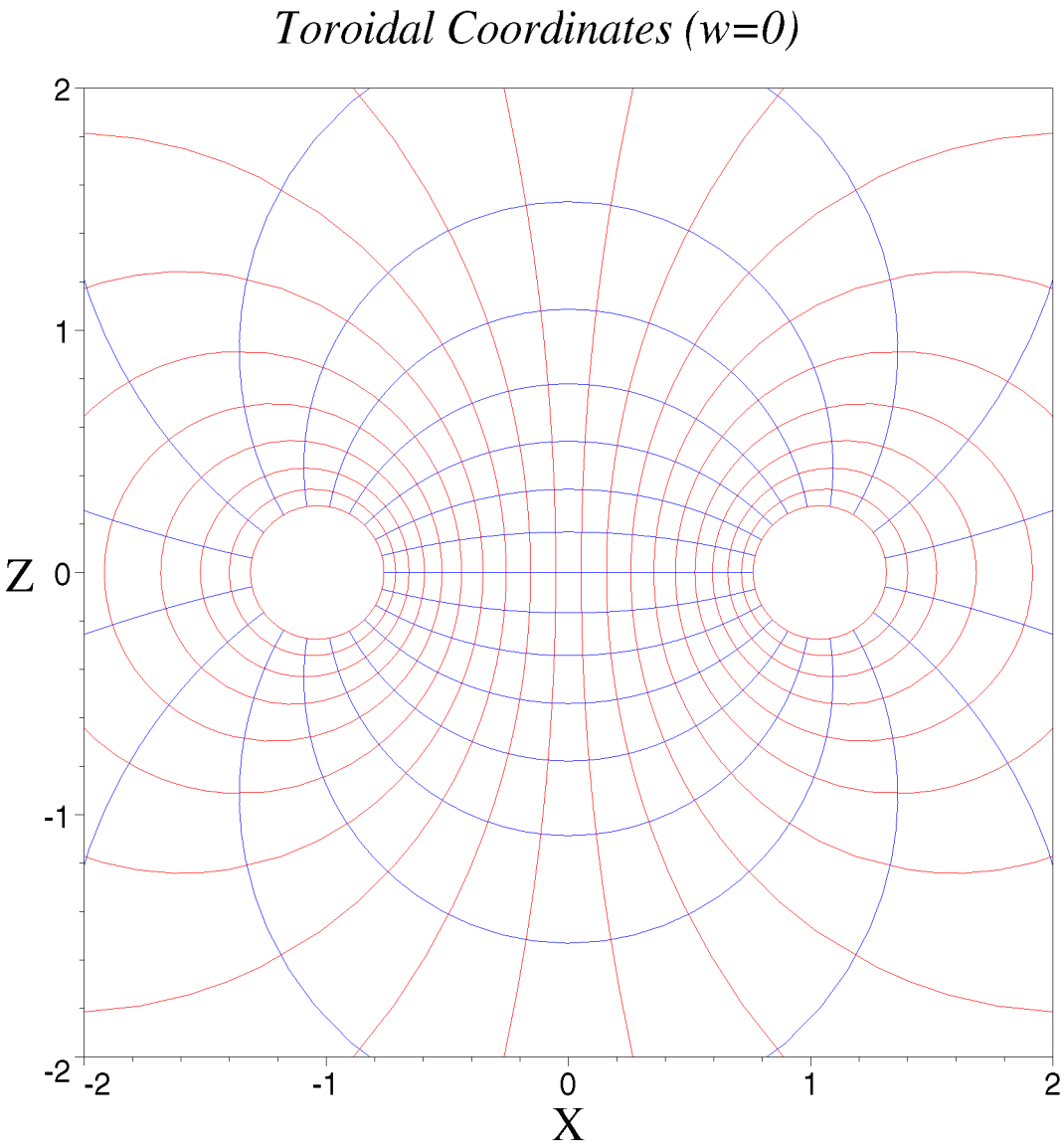
$$\mathrm{coordplot3D}(\mathrm{xyz}(u, v, w), u_0 = 1.2, v_0 = 1.2, w_0 = 0, u = -\pi .. \pi, v = 0 .. 2, w = -\pi .. \pi,$$
$$orientation = [-145, 70], view = [0 .. 2, -2 .. 2, -1 .. 1.6], title = \text{"Toroidal Coordinates"})$$

Toroidal Coordinates



$$xz := \mathrm{fn}\left(\left[\frac{\sinh(v)}{\cosh(v)-\cos(u)}, \frac{\sin(u)}{\cosh(v)-\cos(u)}\right], u, v\right)$$

coordplot2D(xz(u, v), $u = -\pi$ .. π, $v = -2$ .. 2, 20, *view* = [-2 .. 2, -2 .. 2],
title = "Toroidal Coordinates (w=0)", *labels* = ["X", "Z"])



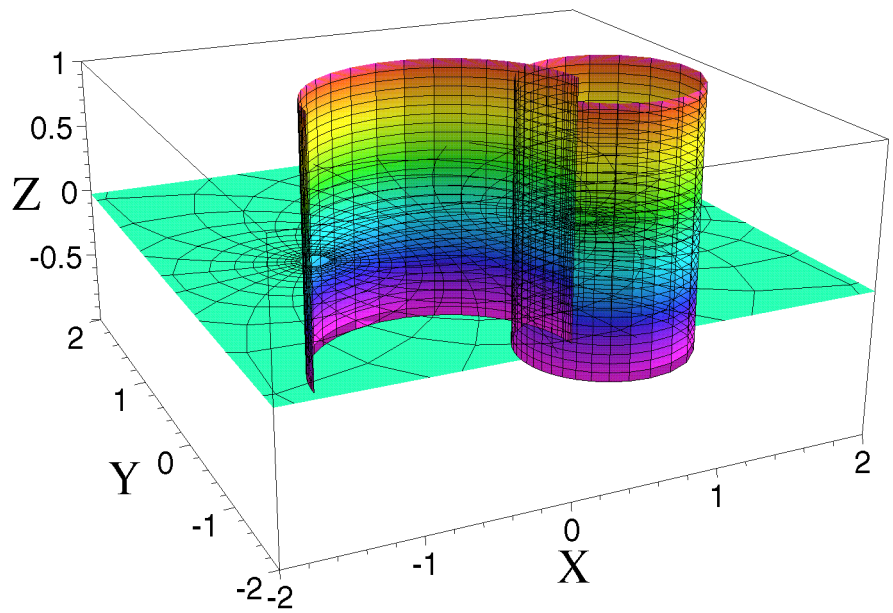
Bipolar Cylindrical

```
bipolarcylindrical:          ( Spiegel )
   x = a*sinh(v)/(cosh(v)-cos(u))
   y = a*sin(u)/(cosh(v)-cos(u))
   z = w
```

$$xyz := \mathrm{fn}\left(\left[\frac{\sinh(v)}{\cosh(v)-\cos(u)}, \frac{\sin(u)}{\cosh(v)-\cos(u)}, w\right], u, v, w\right)$$

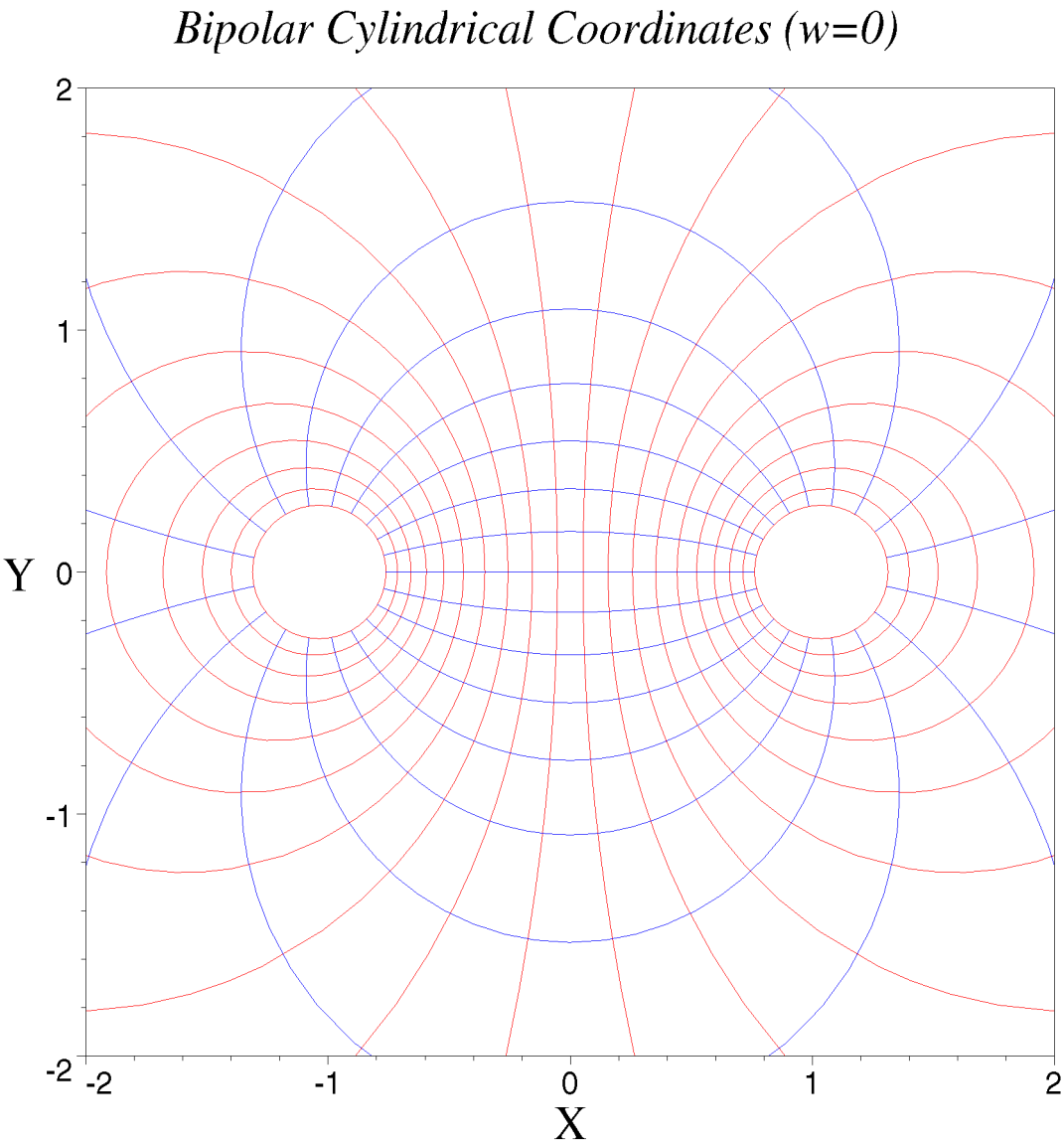
$$\mathrm{coordplot3D}(\mathrm{xyz}(u, v, w), u_0 = 1.5, v_0 = 1.2, w_0 = 0, u = -\pi .. \pi, v = -3 .. 3, w = -1 .. 1,$$
$$orientation = [-115, 70], view = [-2 .. 2, -2 .. 2, -1 .. 1], title = \text{"Bipolar Cylindrical Coordinates"})$$

Bipolar Cylindrical Coordinates



$$xy := \mathrm{fn}\left(\left[\frac{\sinh(v)}{\cosh(v)-\cos(u)}, \frac{\sin(u)}{\cosh(v)-\cos(u)}\right], u, v\right)$$

coordplot2D(xy(u, v), $u = -\pi$ .. π, $v = -2$ .. 2, 20, *view* = [−2 .. 2, −2 .. 2], *title* = "Bipolar Cylindrical Coordinates (w=0)", *labels* = ["X", "Y"])



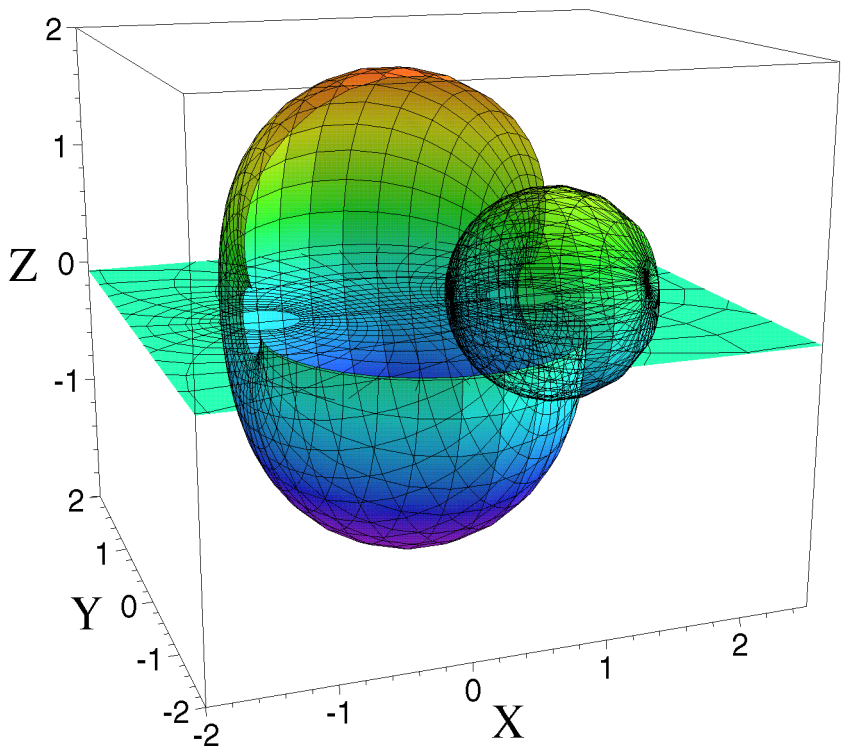
Bispherical

```
bispherical:
  x = sinh(v)/d
  y = sin(u)*cos(w)/d
  z = sin(u)*sin(w)/d                ( where d = cosh(v) - cos(u) )
```

$$xyz := \mathrm{fn}\left(\left[\frac{\sinh(v)}{\cosh(v)-\cos(u)}, \frac{\sin(u)\cos(w)}{\cosh(v)-\cos(u)}, \frac{\sin(u)\sin(w)}{\cosh(v)-\cos(u)}\right], u, v, w\right)$$

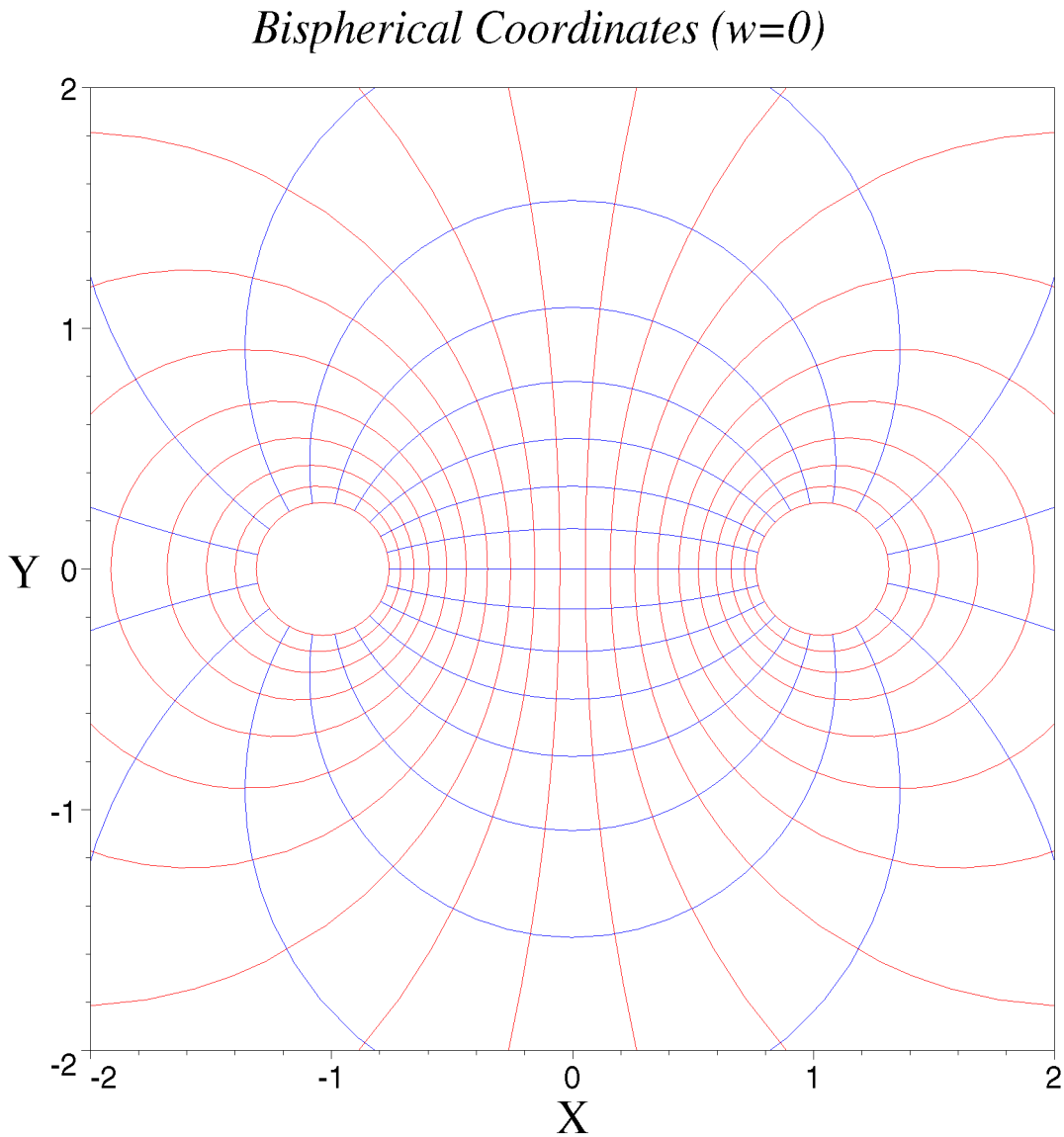
$$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = 1, v_0 = 1, w_0 = 0, u = -\pi .. \pi, v = -2 .. 2, w = -\pi .. \frac{1.1\,\pi}{2}, \right.$$
$$\left. orientation = [-110, 75], view = [-2 .. 2.5, -2 .. 2, -2 .. 2], title = \text{"Bispherical Coordinates"}\right)$$

Bispherical Coordinates



$$xy := \mathrm{fn}\left(\left[\frac{\sinh(v)}{\cosh(v)-\cos(u)}, \frac{\sin(u)}{\cosh(v)-\cos(u)}\right], u, v\right)$$

coordplot2D(xy(u, v), $u = -\pi$ .. π, $v = -2$ .. 2, 20, *view* = [−2 .. 2, −2 .. 2],
title = "Bispherical Coordinates (w=0)", *labels* = ["X", "Y"])



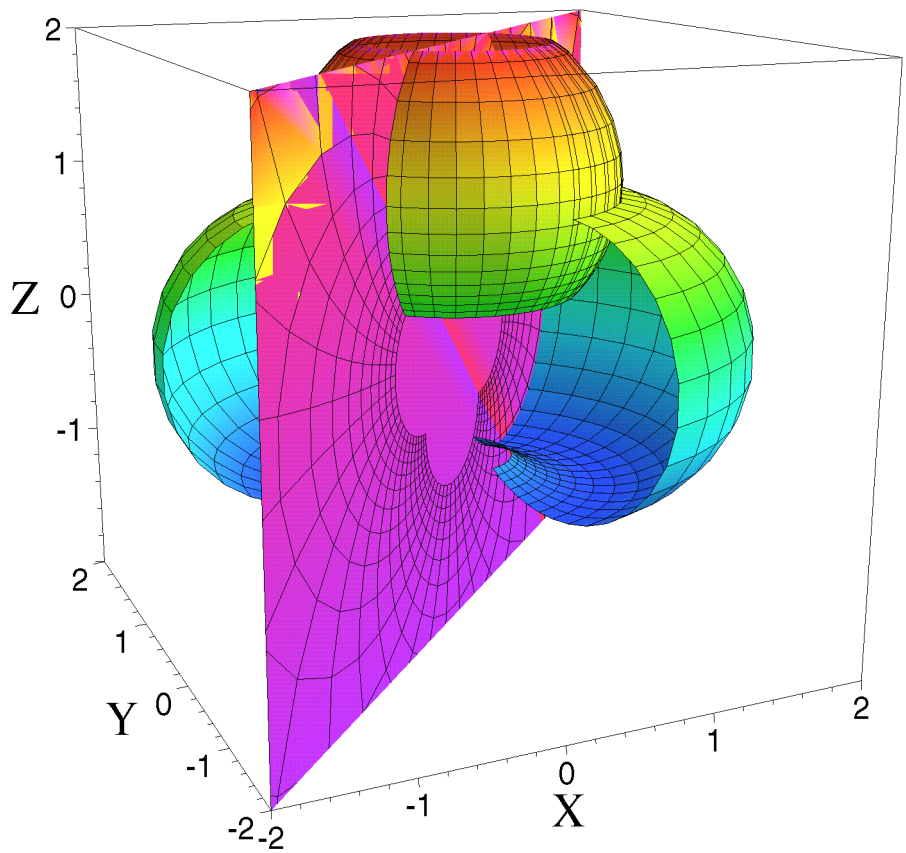
Tangent Spheroidal

```
tangentsphere:
  x = u*cos(w)/(u^2+v^2)
  y = u*sin(w)/(u^2+v^2)
  z = v/(u^2+v^2)
```

$$xyz := \mathrm{fn}\left(\left[\frac{u\cos(w)}{u^2+v^2}, \frac{u\sin(w)}{u^2+v^2}, \frac{v}{u^2+v^2}\right], u, v, w\right)$$

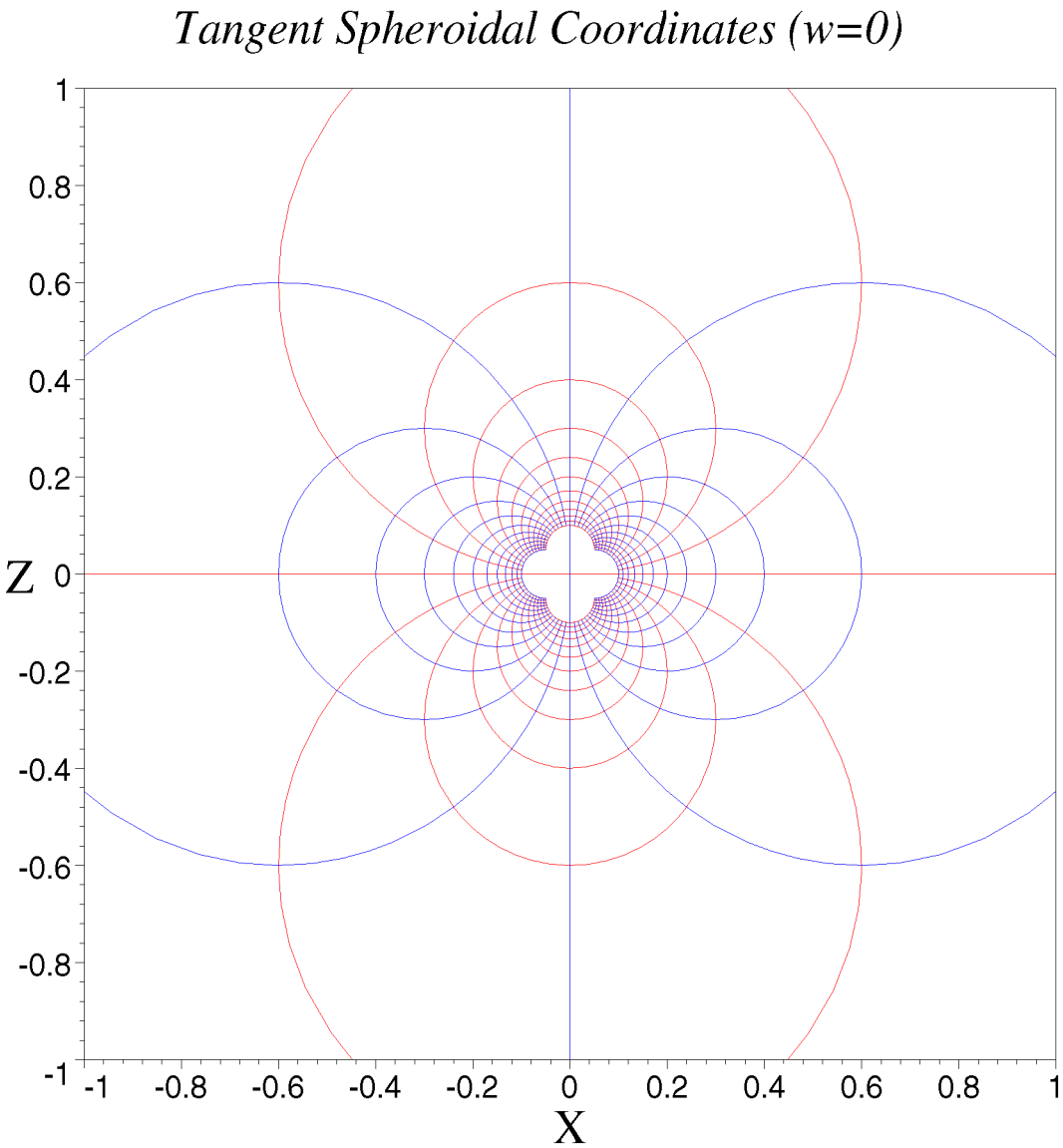
$$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = .5, v_0 = .45, w_0 = \mathrm{evalf}\left(\frac{\pi}{4}\right), u = -1 .. 1, v = -1 .. 1, w = -.4\,\pi .. \pi,\right.$$
$$\left. orientation = [-114, 75], view = [-2 .. 2, -2 .. 2, -2 .. 2], title = \text{"Tangent Spheroidal Coordinates"}\right)$$

Tangent Spheroidal Coordinates



$$xz := \mathrm{fn}\left(\left[\frac{u}{u^2+v^2}, \frac{v}{u^2+v^2}\right], u, v\right)$$

coordplot2D(xz(u, v), $u = -10$ .. 10, $v = -10$ .. 10, 25, *view* = [−1 .. 1, −1 .. 1],
title = "Tangent Spheroidal Coordinates (w=0)", *labels* = ["X", "Z"])



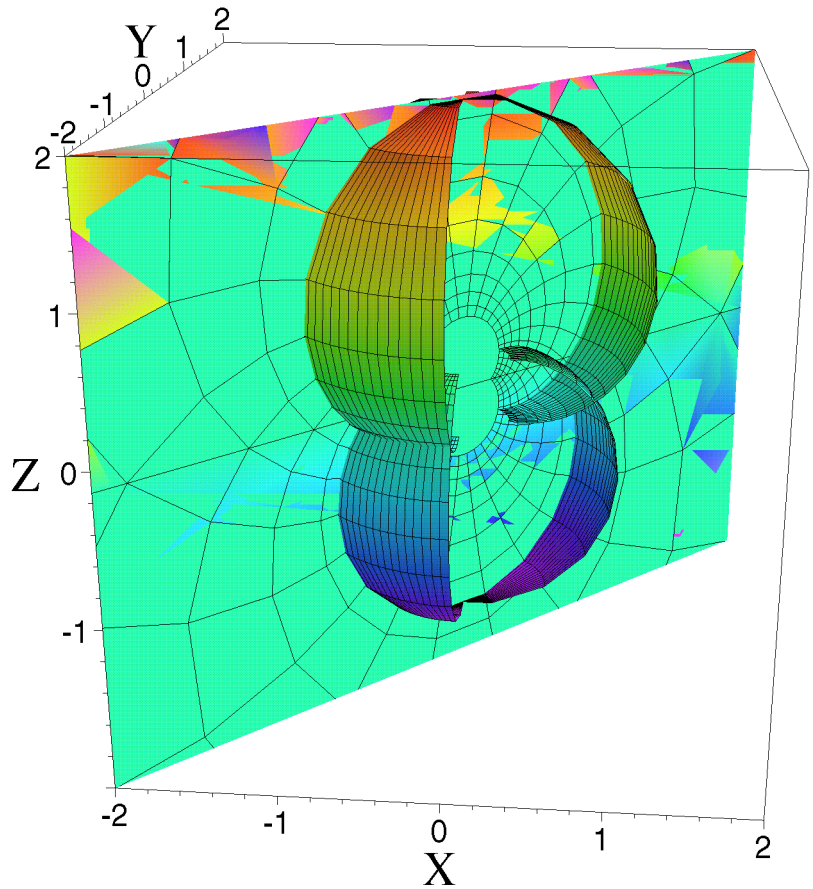
Cardioidal

```
cardiodal:
  x = u*v*cos(w)/(u^2+v^2)^2
  y = u*v*sin(w)/(u^2+v^2)^2
  z = (u^2-v^2)/2/(u^2+v^2)^2
```

$$xyz := \mathrm{fn}\left(\left[\frac{u\,v\cos(w)}{(u^2+v^2)^2}, \frac{u\,v\sin(w)}{(u^2+v^2)^2}, \frac{u^2-v^2}{2\,(u^2+v^2)^2}\right], u, v, w\right)$$

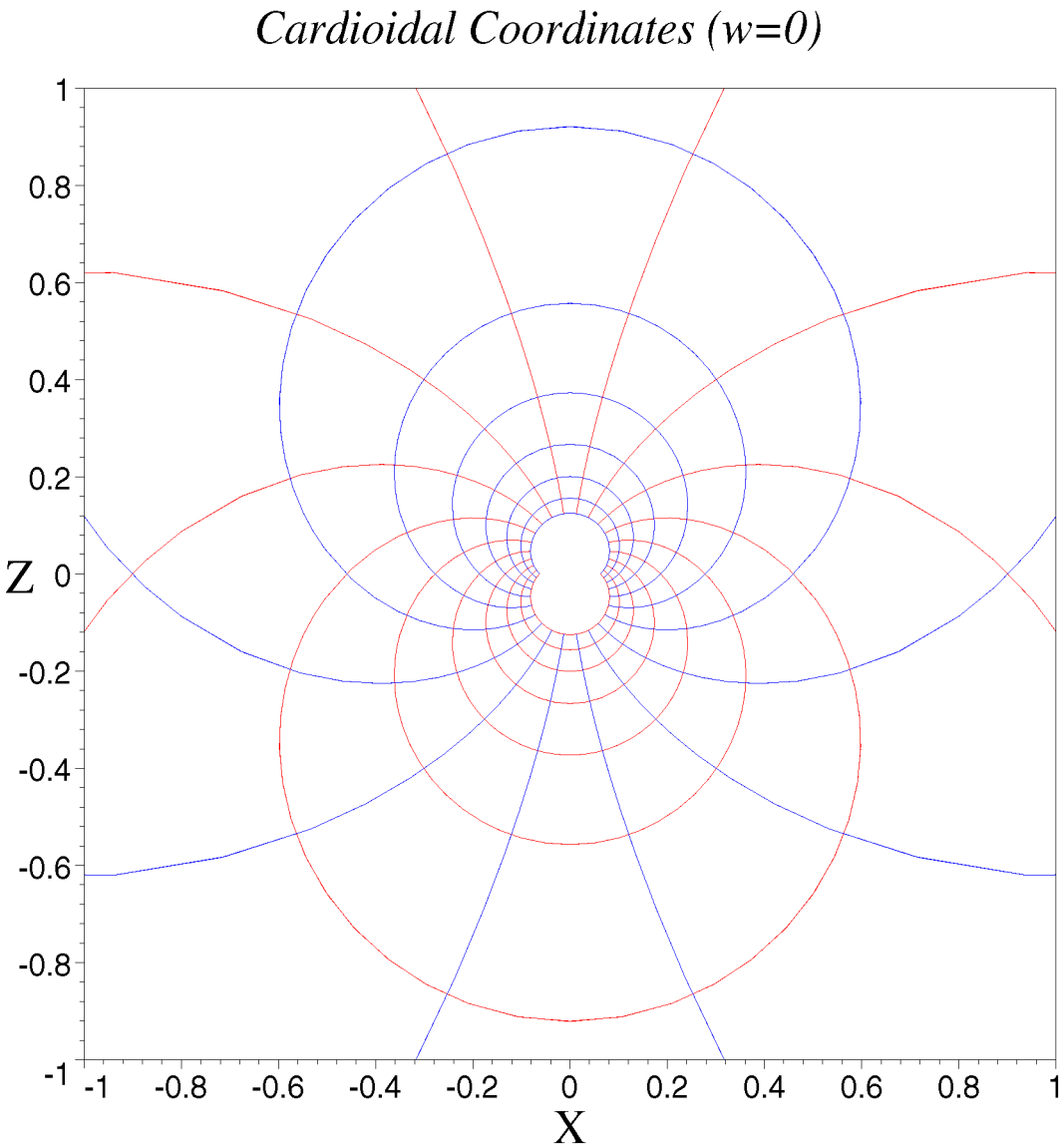
$$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = .5, v_0 = .55, w_0 = \mathrm{evalf}\left(\frac{\pi}{4}\right), u = -1 .. 1, v = -1 .. 1, w = 0 .. \frac{\pi}{2},\right.$$
$$\left. orientation = [-85, 70], view = [-2 .. 2, -2 .. 2, -2 .. 2], title = \text{"Cardioidal Coordinates"}\right)$$

Cardioidal Coordinates



$$xz := \mathrm{fn}\left(\left[\frac{u\,v}{(u^2+v^2)^2}, \frac{u^2-v^2}{2\,(u^2+v^2)^2}\right], u, v\right)$$

coordplot2D(xz(u, v), $u = -2 .. 2$, $v = -2 .. 2$, 20, *view* = [−1 .. 1, −1 .. 1], *title* = "Cardioidal Coordinates (w=0)", *labels* = ["X", "Z"])

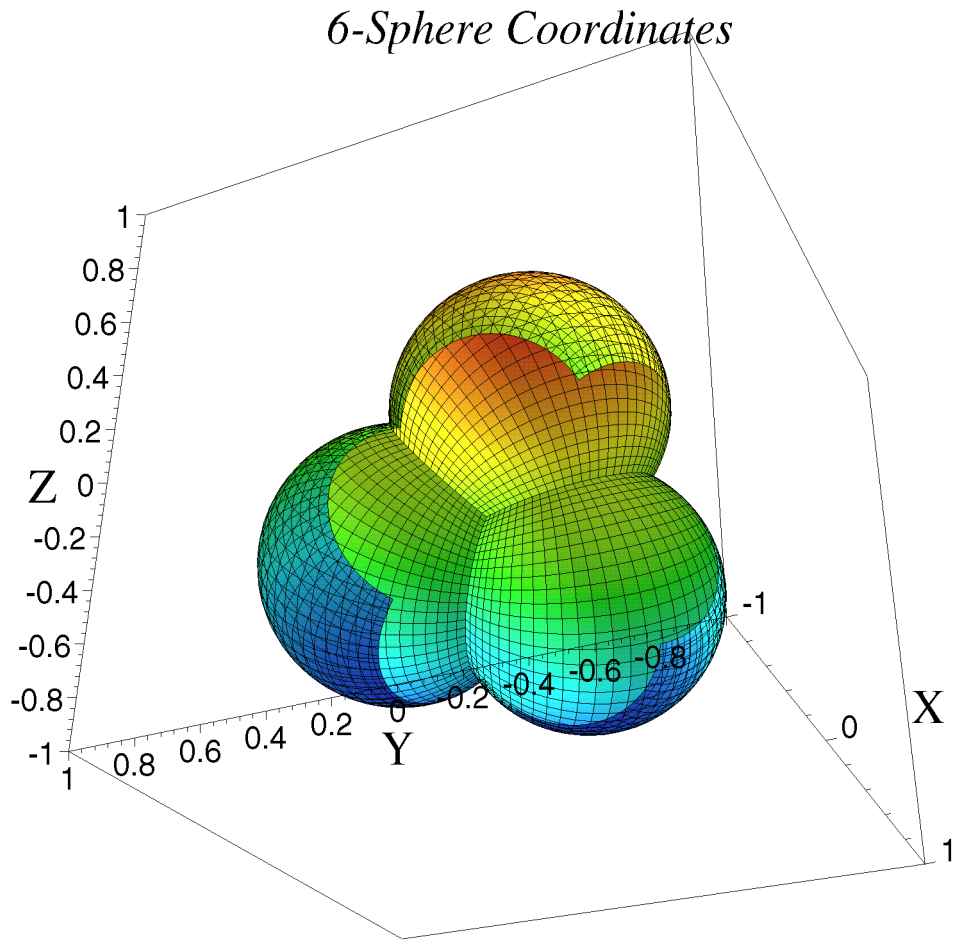


6-Sphere

```
sixsphere:                    (6-sphere)
  x = u/(u^2+v^2+w^2)
  y = v/(u^2+v^2+w^2)
  z = w/(u^2+v^2+w^2)
```

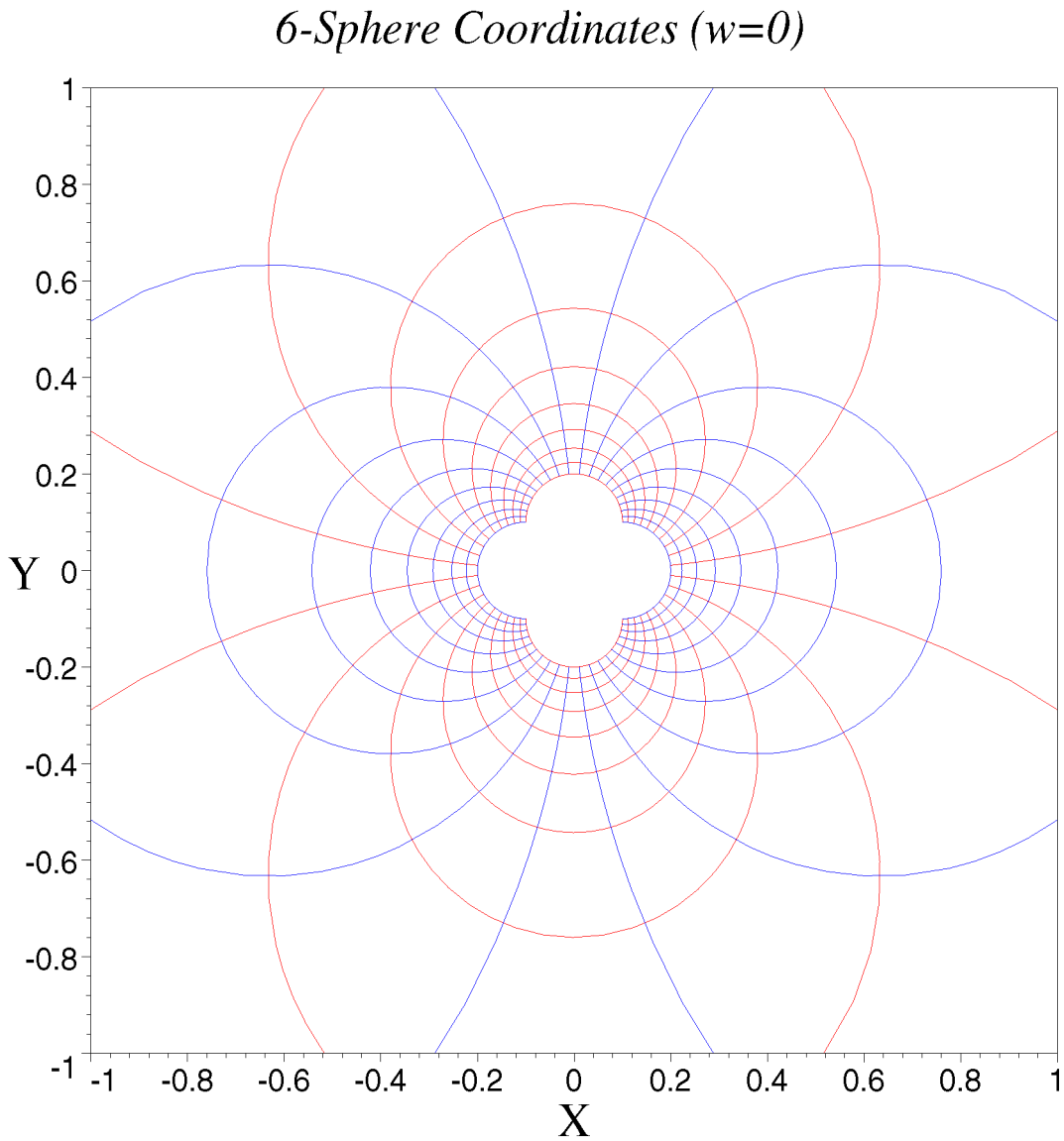
$$xyz := \mathrm{fn}\left(\left[\frac{u}{u^2+v^2+w^2}, \frac{v}{u^2+v^2+w^2}, \frac{w}{u^2+v^2+w^2}\right], u, v, w\right)$$

$$\mathrm{coordplot3D}(\mathrm{xyz}(u, v, w), u_0 = 1, v_0 = 1, w_0 = 1, u = -1 .. 1, v = -1 .. 1, w = -1 .. 1,$$
$$orientation = [-155, 120], view = [-1 .. 1, -1 .. 1, -1 .. 1], title = \text{"6-Sphere Coordinates"})$$



$$xy := \mathrm{fn}\left(\left[\frac{u}{u^2+v^2}, \frac{v}{u^2+v^2}\right], u, v\right)$$

coordplot2D(xy(u, v), $u = -5$ .. 5, $v = -5$ .. 5, 20, *view* = [−1 .. 1, −1 .. 1], *title* = "6-Sphere Coordinates (w=0)", *labels* = ["X", "Y"])



Maxwell Cylindrical

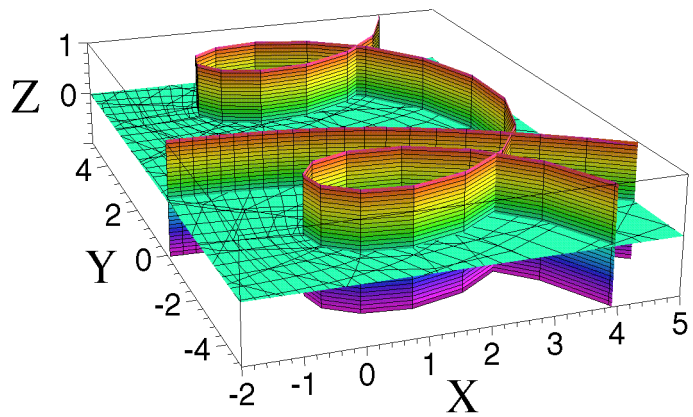
```
maxwellcylindrical:
  x = a/Pi*(u+1+exp(u)*cos(v))
  y = a/Pi*(v+exp(u)*sin(v))
  z = w
```

$xyz := \mathrm{fn}([u + 1 + \mathbf{e}^{u} \cos(v), v + \mathbf{e}^{u} \sin(v), w], u, v, w)$

$\mathrm{coordplot3D}\left(\mathrm{xyz}(u, v, w), u_0 = .8, v_0 = \mathrm{evalf}\left(-\frac{\pi}{4}\right), w_0 = 0, u = -3 .. 3, v = -2\,\pi .. 2\,\pi, w = -1 .. 1,\right.$

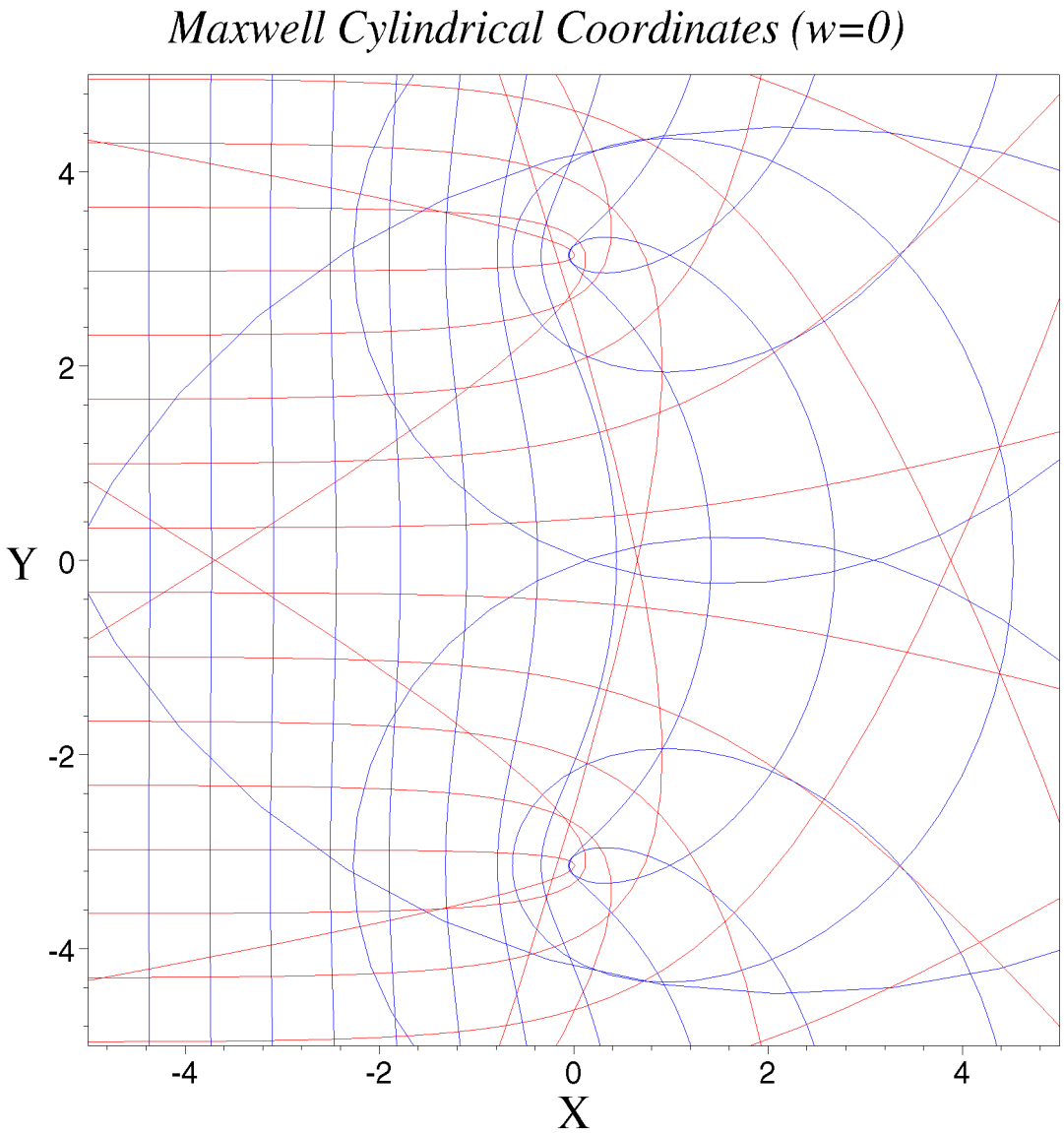
$\left. orientation = [-110, 70], view = [-2 .. 5, -5 .. 5, -1 .. 1], title = \text{"Maxwell Cylindrical Coordinates"}\right)$

Maxwell Cylindrical Coordinates



$xy := \mathrm{fn}([u + 1 + \mathbf{e}^{u} \cos(v), v + \mathbf{e}^{u} \sin(v)], u, v)$

coordplot2D(xy(u, v), $u = -6 .. 6$, $v = -2\,\pi .. 2\,\pi$, 20, *view* = [−5 .. 5, −5 .. 5],
title = "Maxwell Cylindrical Coordinates (w=0)", *labels* = ["X", "Y"])


Maxwell Cylindrical Coordinates (w=0)
Y
X
4
2
0
-2
-4
-4
-2
0
2
4

?